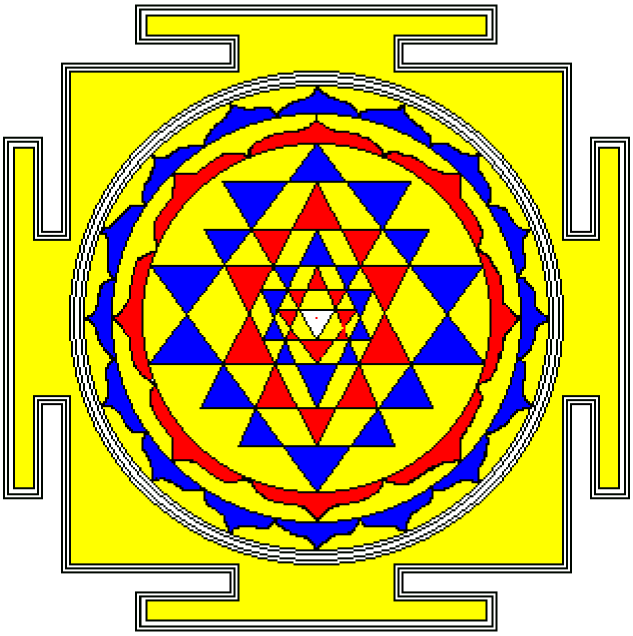
GEOMETRY OF *SRICHAKRA*

(Includes a computer program to calculate and draw *Srichakra*)

Revised Second Edition



T. V. Ananthapadmanabha

GEOMETRY OF *SRICHAKRA*

Revised Second Edition

T. V. Ananthapadmanabha

Publishers

Second Edition *Shravan*, 2020.

Published by the Author

Dr. T. V. Ananthapadmanabha
127, “Anugraha”, Temple Road., 13th Cross
Malleswaram, Bangalore 560 003. Tel: 9886863651
Email: tva.blr@gmail.com

E-Publishing

TO

Sri Matha

Preface to the Second Edition

The first edition of this book with the same title but spelt as 'Geometry of *Sricakra*' was published in 1998 with co-author (late) Sri M N Ramakrishna. In this Second Edition, instead of transliteration convention of the first edition, Sanskrit words are spelt in English for easy readability. The first edition has been well received by the readers. A few minor mistakes found in the first edition have been corrected. A major change in this revised Second Edition is to relate the concept of invariance in *Srichakra* to 'Golden Ratio' or 'Divine Proportion' and add Chapter 6 on constructing *Srichakra* based on 'Two Circles Hypothesis' proposed by the author.

I hope this work will appeal to the ardent students interested in the Indian mysticism, in general, and *Srichakra*, in particular.

Bangalore
Shravan, 2020

Ananthapadmanabha

PREFACE of the First Edition

(Reproduced)

SRICAKRA* is a *yantra* (*mandala*). A *yantra* or *mandala* is a geometrical drawing used for worship. *Sricakra* is a widely used *yantra*. *Sricakra* is highly esoteric and embodies in itself deep philosophical and yogic concepts.

During a course of literature survey we came across several drawings of *Sricakra*, widely differing from one another. Naturally, several questions arise: 'Are all these drawings valid representations of *Sricakra*?' 'If not, which one of them is valid?' 'How do you determine the validity?'

Further, several different procedures for constructing *Sricakra* have been described in literature. When these procedures are implemented on a computer, errors have been noticed. On seeking the details of construction of *Sricakra* from learned scholars we obtained only a qualitative description. A Russian author who in his paper on *Sricakra* considers the problem to be highly complex. We found an article written by Sri M. P. Shankaranarayanan (MPS) extremely useful but not complete.

Sricakra is a clearly defined geometrical figure. Despite its importance it is surprising that a satisfactory procedure to draw Sricakra is not easily found in available literature. This prompted us to work on the details of the geometry of *Sricakra*. We have used MPS's article as a starting point in our investigations. The spritual significance or ritualistic aspects of *Sricakra* are not discussed in this book.

We are presenting the outcome of our efforts so that persons interested in the *Sricakra* drawing are benefited. The correct procedure to draw *Sricakra* may be followed so that an accurate drawing can be used for worship. This book also facilitate the devotees of *Sricakra* to become aware of methods of examining its accuracy. We trust this work will motivate others to probe deeper into the fascinating mystical drawing.

Bangalore — *Ananthapadmanabha*

Navaratri, 1998 — *Ramakrishna*

(* In the first edition, transliteration system was followed and hence the spelling 'Sricakra'.)

About the Book

Srichakra is a mystical *yantra* (*mandala*) that embodies in itself deep philosophical and yogic concepts. *Srichakra* is a clearly defined geometrical figure. Yet, it is surprising that a satisfactory procedure for the construction of *Srichakra* is not easily found in the available literature. In this book, we present a clearly defined and well illustrated procedure for drawing *Srichakra*. This book facilitates devotees of *Srichakra* to become aware of the methods of examining its accuracy and also enables them to construct an accurate drawing of *Srichakra*. A new concept called 'Two Circles Hypothesis' for constructing *Srichakra* has been proposed. A computer program listing to calculate and plot *Srichakra* is also included. This book may motivate others to probe deeper into the geometry of the fascinating mystical drawing.

About the Author

T. V. Ananthapadmanabha holds a PhD degree in electrical communication engineering from a reputed institute in Bangalore, India. He has researched in world renowned laboratories in the area of speech science and has published a large number of technical papers in peer reviewed international journals. Apart from his academic pursuit, as an entrepreneur he has produced software products in the area of voice and speech catering to speech and hearing impaired. His interest also lies in understanding the contribution of Indian *rishis*. He has also authored the work, *Inner Workings During Yoga Practice,* which is based on the teachings of his spiritual guide, Sri Srinivasa Ranga (or in short, Sriranga) Sadguru. For more details on Sri Sriranga sadguru, visit www.ayvm.in.

CONTENTS

1
Description and Definition of Terms

1. 1. Parts of *Srichakra*

Srichakra is composed of nine *chakra*s or nine enclosures (*nava-avarana*). From a geometrical point of view, these nine *chakra*s can be divided broadly into three groups:

(a) *Bhupura*

(b) *Trivalaya*, *Shodasha-dala Padma*, *Ashta-dala Padma* and

(c) Inner-most circle consisting of forty-three overlapping triangles.

The drawing belonging to group (c) is by itself popularly known as *Srichakra*. A brief description as well as the geometrical details of groups (a) and (b) are given in this Chapter. Only a brief description of group (c) is given in this Chapter and the rest of the book deals with the detailed geometrical aspects related to group (c).

1. 2. *Bhupura* : This is also referred to as *Bhugrha*. This is in the form of a square called *Chaturashra*. It is drawn with three parallel lines. It is considered as the first *chakra, Trailokya-mohana*. The three lines symbolise the feet, the knee and the thigh of *Srichakra* personified. There are four gates or portals at the middle of each of the four sides. *Bhupura* is drawn in different styles. In some drawings the gate is shown open and without projection (Fig.1.1a) while in others the gate is shown open but with projection (Fig.1.1b). In the third kind the gate projects out of the square and is closed (Fig.1.1c). The drawing shown in Fig.1.1c resembles a tortoise (*kurma*).

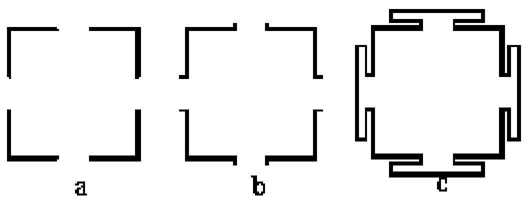


Fig.1.1. Three representations of Bhupura.

After a careful study, we have adopted the dimensions for *Bhupura* as shown in Fig.1.2. We use a diameter of 108 units for the inner-most circle. Value of the unit depends on the size of *Srichakra* to be drawn. For example, one unit could be one milli-meter for the inner-most circle of 108 mm size or one unit could be one-tenths of an inch for the inmost-circle of 10.8 inch size. *Bhupura* will then be enclosed in a circle of diameter 224 units. The circle of diameter of 224 units is shown only for the purpose of explaining the geometry of *Bhupura*. In the actual drawing it has to be left out as in Fig.1.1c.

The number 224 corresponds to the total number of *Bhuvanas* and the number 108 corresponds to the number of *Bhuvanas* of *Prihtvi Tatva* (*Elements of Hindu Iconography* by T. A. Gopinath Rao). The technical reason for selecting a diameter of 108 units for the inner-most circle is presented in Sec.3. 5.

The three parallel lines in *Bhupura* are called the outer, middle and inner lines. The three outer corners on the circle in the upper right hand quadrant lie on imaginary lines at 30, 45 and 60 degrees to the horizontal (Fig.1.2). Thus the twelve outermost corners of *Bhupura* as well as the size of outer square are determined. Each side of the inner square is tangential to the outermost circle of *Trivalaya*. See Sec. 1.3 for the size of *Trivalaya*. The middle line of the square is drawn at the middle of outer and inner lines. Projection of the gate is such that imaginary lines at 15 and 75 degrees to the horizontal pass through the inner corners of the gates as shown in Fig.1.2. The opening of the gate comes out to be approximately one third the side of the square.

1. 3. *Trivalaya* : This is also known as *Mekhala-traya*. This consists of the triad of three concentric circles immediately inside the square with the same centre as that of the inner-most circle. Note that *Trivalaya* is not considered as one of the nine *chakras*. Some authors consider it as a part of *Bhupura*. *Trivalaya* is not seen in the *Srichakra* drawing of *Hayagriva* convention.

Diameter of the outermost circle of *Trivalaya* is square root of 2 (1.414) times 108 units. In other words diameter of the outermost circle of *Trivalaya* is the diagonal of the square with side 108. This comes out to be 152.735, but shown as 153 in Fig.1.2. The inmost circle of *Trivalaya* is one and one-third (4/3) times 108, i.e., 144 units. The two intermediate circles are equally spaced between the outer-most and inmost circles (Fig.1.2).

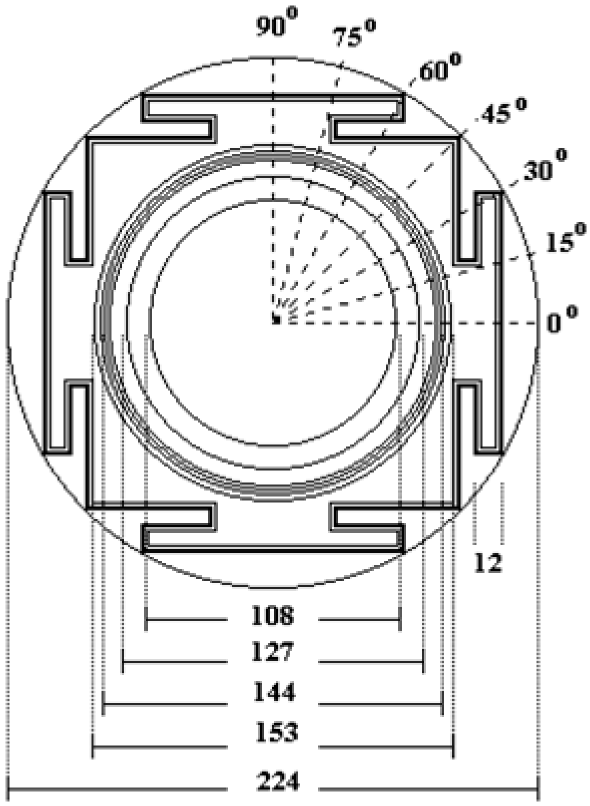


Fig.1.2. Geometry of Bhpura and Trivalaya

1. 4. *Shodasha-dala Padma* : (Circle with 16 petals).

Immediately inside the *Trivalaya* is an annular ring with sixteen petals. This is considered as the second *chakra*, *Sarva-asha-paripuraka*. Petals are all of equal size. It is insisted that there should not be any gap (*kesara*) between two adjacent petals.

1. 5. *Ashta-dala Padma* : (Circle with 8 petals):

Between the *Shodasha-dala Padma* and the inner-most circle is an annular ring with eight petals constituting the third *chakra, Sarva-samksobhana*. Petals are all of equal size. It is insisted that there should not be any gap (*kesara*) between two adjacent petals.

Circles with 16 and 8 petals have diameters of 144 and 127 units respectively compared to the inner-most circle of diameter of 108 units. (See Fig.1.2). This will ensure nearly equal area for the two annular rings. Also, we have drawn the petals in such a way that they may be interpreted as either pointing inwards or outwards (See Fig.1.3,).

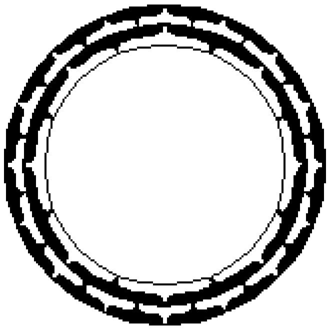


Fig.1.3. Shodasha-dala Padma (petals in the outer ring) and Ashta-dala Padma (petals in the inner ring)

1. 6. Inner-most Circle : This is the most important part of *Srichakra* and also technically the most involved drawing. It is a configuration of nine triangles, four upright and five inverted, overlapping to form the main *chakra* consisting of forty-three small triangles. All these triangles are within the inner-most circle. The forty-three triangles complex is also referred to as *tri-chatvarimshat-kona*.

The inner-most circle consists of the following components.

(i) ***Chatur-dashara*** or A Group of 14 triangles: (Fig.1.4a). This is the fourth *chakra* called *Sarva-sowbhagya-dayaka*.

(ii) ***Bahya-dashara*** or A Group of 10 outer triangles: (Fig.1.4b). This is the fifth *chakra* called *Sarvartha-sadhaka*.

(iii) ***Antar-dashara*** or A Group of 10 inner triangles: (Fig.1.4c). This is the sixth *chakra* called *Sarva-rakshaakara*.

(iv) ***Ashtakona*** or A Group of 8 triangles: (Fig.1.4d). This is the seventh *chakra* called *Sarva-rogahara*.

(v) **Innermost Triangle**: The eighth *chakra* is the innermost inverted triangle (Fig.1.4e) called *Sarva-siddhi-prada*.

(vi) ***Bindu***: The ninth *chakra* is the *Bindu* (literally, 'point') that is considered to be a triangle of zero size (Fig.1.4f). This *chakra* is called *Sarva-ananda-maya*. This coincides with the centre of the circle. In some drawings we have found *Bindu* displaced from the centre and placed just above the base of the innermost triangle.

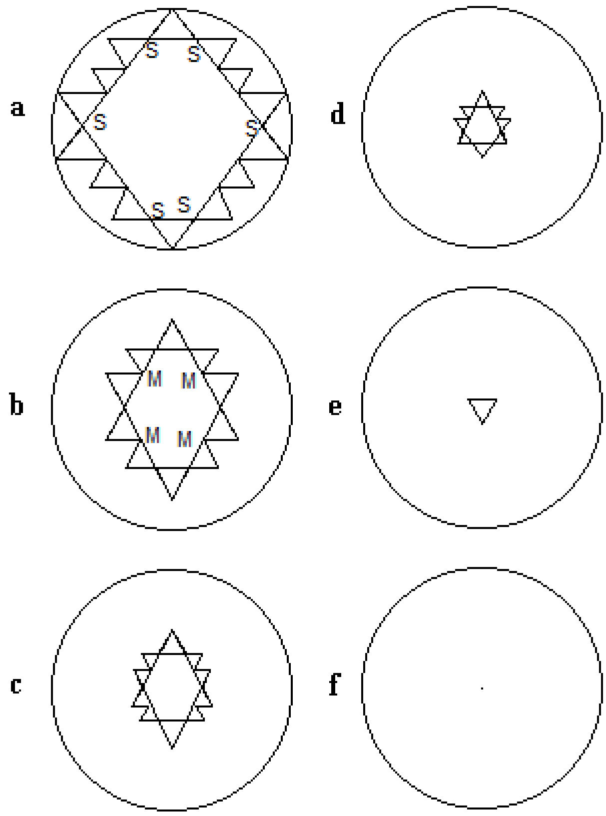


Fig.1.4. Six groups of 43 triangles: (a) Group of 14 triangles, Chatur-dashara; Sandhi is marked as S (b) Outer Group of 10 triangles Bahya-dashra; Marma is marked as M (c) Inner Group of 10 triangles Antar-dashara (d) Group of 8 triangles, Ashtakona (e) Inner-most inverted triangles, Yoni and (f) Triangle of ZERO dimensions, Bindu,

1. 7. Assignment of Colours

Parts of *Srichakra* have been assigned colours. The original reference for this is not known. It is said to be traditional. Based on secondary sources, the colours assigned for parts of *Srichakra* are as follows. The three lines of *Bhupura* from outer to inner have the colours white, red and yellow respectively according to one source. According to another, the three lines are assigned the colours dark green, light green and dark green respectively. The space between *Bhupura* and *Trivalaya* is assigned the colour yellow. The annular spaces between the circles of *Trivalaya* are assigned the colours dark green and light green alternately. *Shodasha-dala padma*, *Ashta-dala padma*, *Chatur-dashara*, *Bahya-dashara*, *Antar-dashara* and *Ashtakona* have been assigned blue and red colours alternately. The innermost triangle (yoni) is assigned the colour white. The colour for the *Bindu* is not specified. We guess that it is red.

1. 8. Mode of Depicting *Srichakra*: Usually all the nine *chakras* are drawn on the same horizontal plane, on a metal plate of copper or bronze or silver. This is called *Bhu-prashtara* representation. Another form is the so called *Meru-prashtara*, where the nine *chakras* are at different heights. It is to be appreciated that *Bhu-prashtara* is the projection of *Meru-prashtara* on a horizontal plane. The *Srichakra* with the alphabets of Sanskrit language assigned to various parts as per tradition is known as *Kailasa-prashtara*.

1. 9. Definition of Terms

Trikona or Kona: A triangle. This is of two types. (i) *Vahni Kona:* An upward triangle with base below and apex above (ii) *Yoni*: An inverted triangle with base at above and apex below.

Sandhi is the point of intersection of two lines (*dvi-rekha sangama*).

Marma is the point of intersection of three lines (*tri-rekha sangama*).

There are six *Sandhi*-s in each of the *chakras* 4 to 7 and thus a total of 24 *Sandhi*s. Recall that the *chakras* 4 to 7 are *Chatur-dashara*, *Bahya*-dashara, Antar-*dashara* and *Ashtakona*. *Sandhi*s. These can easily be located in Figs.1.4a to 1.4d.

In case of *Marma*s, the mere junction of three lines is not taken into account. Only the point through which three lines cross is considered a *Marma*. The number of *Marma*s common to two consecutive *chakras* are shown in the following Table. Umanandanatha in his work, *Nityotsava* mentions 24 *Sandhi*s and 18 *Marma*s.

There is a divergence of opinion concerning the number of *Marma*s. Some mention the number of *Marma*s to be 24. In some texts 28 *Marma*s are also mentioned. We do not know how these numbers have been arrived at.

Chakras	Number of *Marmas*
4th & 5th	8
5th & 6th	4
6th & 7th	4
7th & 8th	2
Total	**18**

1. 10. Errors

This section restricts its discussion to the errors in the drawing of the forty-three triangle complex in the inner-most circle. There are two kinds of errors:

(a) Fine Errors - arising due to the complexity of the drawing.

(b) Gross Errors - arising due to carelessness on the part of the draftsman or an incomplete understanding of the geometry of the drawing.

Errors of kind (a) are excusable as they may arise because of practical difficulties. In fact this book is mainly concerned with the complexity of the geometry of *Srichakra*. But errors of kind (b) are to be avoided. It is surprising to note that errors of kind (b) are often encountered amongst published drawings. Some even adorn the cover page of books on *Srichakra*! We want to point out the existence of these gross errors so that a person interested in the geometry of *Srichakra* can discriminate between an accurate drawing from an inaccurate one. We shall present examples of errors of kind (b). Some errors are too obvious. On the other hand, some errors are noticeable only on magnification, i.e., when seen through a lens of magnification of two or higher. These errors are best illustrated with the help of sketches.

In Fig.1.5, the errors are shown exaggerated so that the reader may know where to look for them in published drawings. Fig.1.5a shows the case where the apex of a triangle goes beyond the circumference of a circle or a horizontal line. In such cases the triangle is shown truncated. Fig.1.5b shows the case in which the apex is just below the circle or a horizontal line. In some cases the apex either goes beyond a sloping line or falls just short of a sloping line as shown in Fig.1.5c. In case the apex goes beyond the sloping line it is shown truncated.

Figs.1.5d to 1.5f show possible errors in *Sandhi* (intersection of two lines). In Fig.1.5d, slope of the inclined line changes at the intersection. In Fig.1.5e, horizontal lines to the left and right of the intersection are displaced with respect to each other. A thick line might have been drawn to cover-up the error. Fig.1.5f shows the case of an inclined line which retains the same slope but is displaced on either side of the intersection.

Figs.1.5g - 1.5i shows cases of errors in *Marma*. These are similar to errors in *Sandhi*. In particular, Fig.1.5i shows an error where three lines form a small triangle in place of a *Marma*.

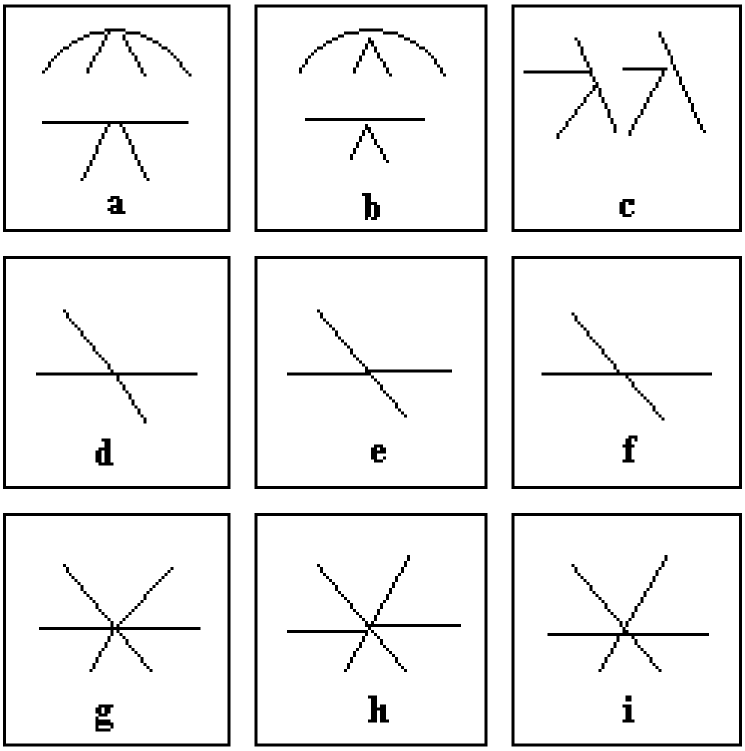


Fig. 1.5. Illustration of errors that may arise in an inaccurate drawing of *Srichakra*

2

Construction of *Srichakra*

2. 1. Nomenclature

In this Chapter we describe a step by step procedure for constructing *Srichakra* (forty-three triangles complex). An understanding of the procedure is a prerequisite for a discussion to be presented in subsequent Chapters. The details of data to be used as input are discussed in the next Chapter. Since *Srichakra* has a symmetry with respect to the vertical axis, the procedure for construction of only the right-half is presented. The left-half is drawn as a mirror image of the right-half.

Fig.2.1 shows the nomenclature of some selected points of the forty-three triangles complex. Points on the left-half are designated with a prime. Thus, for example, point E on the right-half has a corresponding point E' on the left-half. Fig.2.2 shows a part of *Srichakra* to an enlarged scale along with the nomenclature. We have adopted the designations used by Sri M. P. Shankaranarayanan (MPS) and added a few more of our own. This is to facilitate the readers to refer our work in relation to MPS's work (See Sec. 3.3).

It is suggested to the reader to construct *Srichakra* while reading the step by step procedure given below. Traditional data (See Sec. 3.1) have been used for the purpose of explaining the procedure. Diameter AD is taken as 48 units as per the traditional data. The input data to be used are as follows:

AZ = 6 units.	AF=17 units.	AY=27 units.	AC=30 units.
AX=42 units.	AD=48 units.		

What is a unit? The value to be assigned to a 'Unit' depends on the size of the drawing. Thus for example, if a diameter of 6 inches and a scale marked in eighths of an inch are chosen then one unit becomes one-eighths of an inch. Alternately, if a diameter of 48 mm is chosen, one unit becomes 1 mm. For a diameter of 96 mm one unit will be 2 mm.

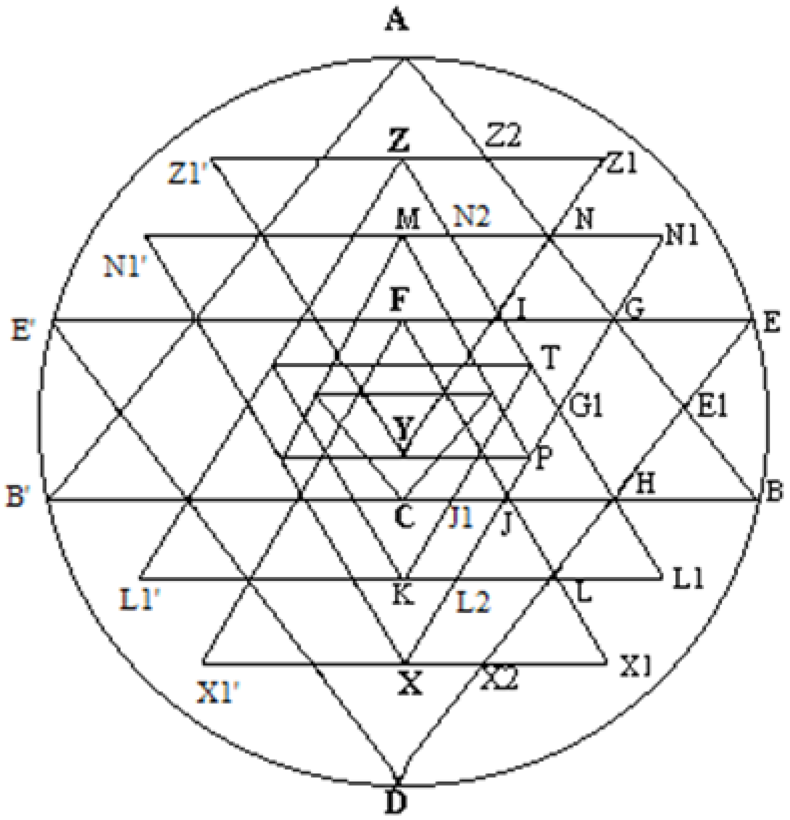


Fig.2.1. Nomenclature used to explain the construction and geometry of Srichakra.

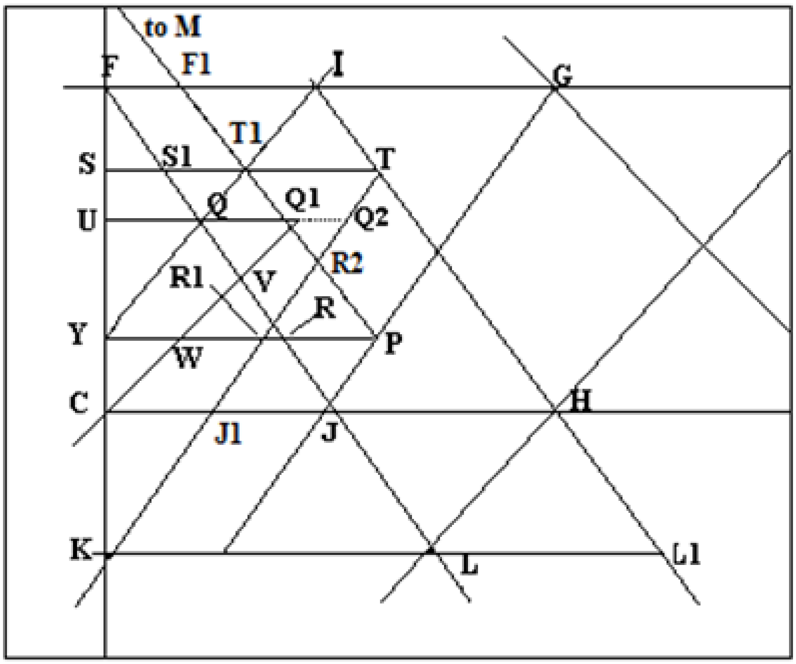


Fig.2. 2. Enlarged view of a part of Srichakra to show the nomenclature.

2. 2 Step by Step Procedure

The procedure is illustrated in Figs. 2.3a to 2.3r. In the following description, points given as input data are shown in bold face and points derived are shown in bold italics.

Fig.2.3a:

(1) Draw a circle of the desired diameter **AD**.

(2) On the vertical central diameter mark the points, **Z, F, Y, C and X**. The lengths AZ, AF, AY, AC and AX (five measurements or five input data points) are given.

Fig.2.3b:

(3) Draw a horizontal from **F** to meet the circle at ***E***.

(4) Draw a horizontal from **C** to meet the circle at ***B***.

(5) Join **A*B***. **A*B*** intersects **F*E*** at ***G***.

(6) Join **D*E***. **D*E*** intersects **C*B*** at ***H***.

Fig.2.3c:

(7) Join **Z*H***. **Z*H*** intersects **F*E*** at ***I***.

Fig.2.3d:

(8) Join **Y*I***.

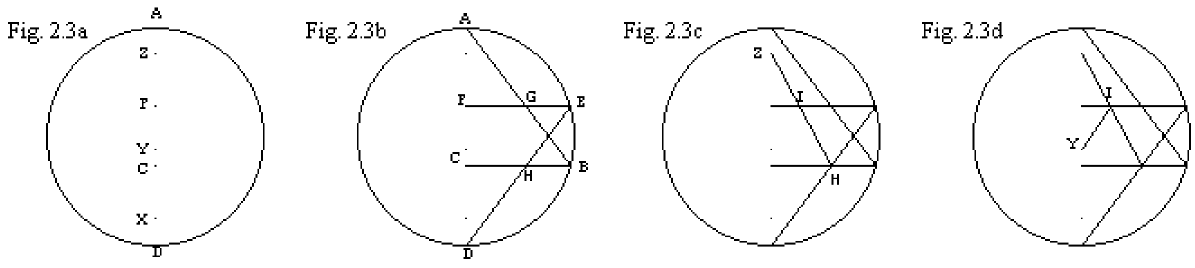


Fig.2.3e:

(9a) Draw a Horizontal from **Z**.

Fig.2.3f:

(9b) Produce **Y*I*** to intersect the horizontal from **Z** at ***Z1***. Erase extra length, if any, beyond ***Z1***.

(10)**Y*Z1*** intersects **A*B*** at ***N.***

Fig.2.3g:

(11) Join **X*G***. **X*G*** intersects **C*B*** at ***J***.

Fig.2.3h:

(12) Draw a horizontal from ***N*** to meet **AD** at ***M***.

(13) Produce **X*G*** to intersect horizontal from ***M*** at ***N1***. Erase extra length, if any, beyond ***N1***.

Fig. 2.3e

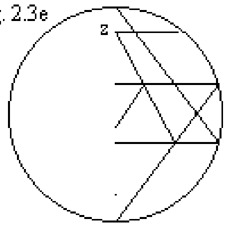


Fig. 2.3f

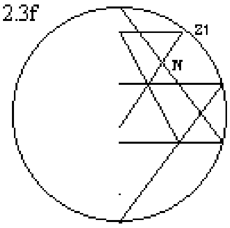


Fig. 2.3g

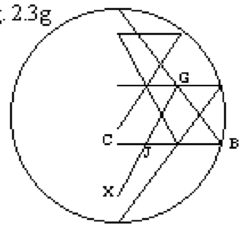


Fig. 2.3h

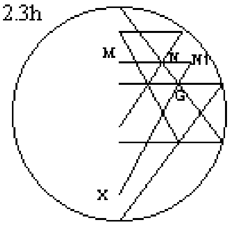


Fig.2.3i:

(14) Join **F*J***. Produce **F*J*** to intersect the horizontal from **X** at ***X1***. Erase extra length, if any, beyond ***X1***.

(15) **F*X1*** intersects **D*E*** at ***L***

(16) **F*X1*** intersects **Y*Z1*** at ***Q***.

Fig.2.3j:

(17) Draw a horizontal from ***L*** to meet **AD** at ***K***.

Fig.2.3k:

(18) Produce **Z*H*** to intersect ***KL*** produced at ***L1.*** Erase extra length, if any, beyond ***L1***.

Fig.2.3l:

(19) Horizontal from **Y** meets **X*G*** at ***P***.

(20) **Y*P*** meets **F*X1*** at ***R***..

Fig. 2.3i

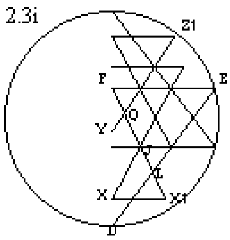


Fig. 2.3j

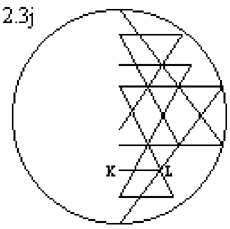


Fig. 2.3k

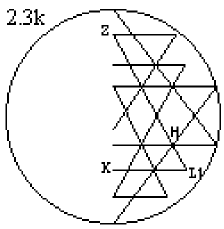


Fig. 2.31

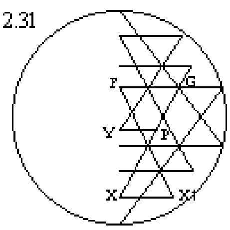


Fig.2.3m:

(21) Join ***MP***. ***MP*** intersects **Y*Z1*** at ***T1***.

Fig.2.3n:

(22) Horizontal from ***Q*** meets **AD** at **U**.

Note: ***Q*** is the point of intersection of **F***X1* and **Y*Z1*** determined in Step 16.

(23) Extend ***UQ*** till ***MP*** to obtain ***Q1***.

Fig.2.3o:

(24) Join **C*Q1***.

Fig.2.3p:

(25) Draw a horizontal from ***T1*** to meet **AD** at ***S***.

Note: ***T1*** is the point of intersection of ***MP*** and **Y*Z1*** determined in Step 21 (See Fig.2.3m).

(26) Produce ***ST1*** to meet **Z*L1*** at ***T***.

Fig. 2.3m

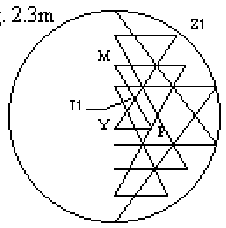


Fig. 2.3n

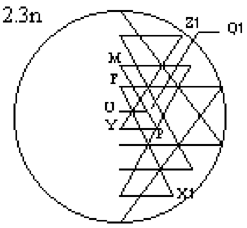


Fig. 2.3o

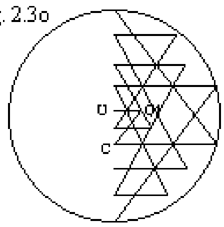


Fig. 2.3p

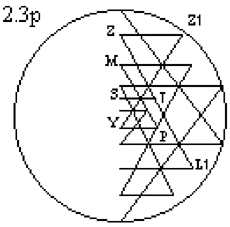


Fig.2.3q:

(27) Join ***TK***.

Fig.2.3r:

(28) Produce a mirror image on the vertical axis to obtain the complete *Srichakra*.

Fig. 2.3q

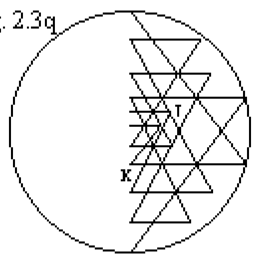
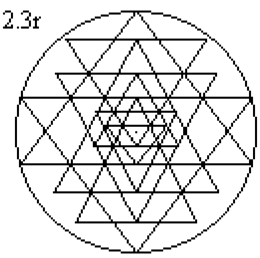


Fig. 2.3r



2. 4. Closing Error

TK of Step 27 is supposed to meet YP at ***R***. But ***R*** is already determined in Step 20 (YP meets FX1 at R, Fig.2.3l). Note that ***R*** is a *Marma* as it is a point of intersection of **F*X1***, ***TK*** and **Y*P***. Let ***TK*** intersect **F*X1*** at ***R1*** **(**See Fig.2.2**)**. Is ***R1 = R***? That is, are they coincident? If not, we refer to the error as the **closing error**. The **closing error** is the distance between ***R1*** and ***R*,**

Closing Error = **Distance(R, R1)**.

This distance depends on the choice of AD, the diameter of the circle. In order to make the closing error independent of the size of the diameter AD, the closing error is **normalized** with respect to AD and expressed as a percentage:

Normalized Closing Error = 100 [Distance(R, R1) / AD]

If the input data are accurate and the construction of *Srichakra* has been carried out correctly, then the closing error will be ideally zero or practically very small.

2. 5. Data: Independent and Derived

All data are relative to the diameter of the circle. The diameter is chosen depending on the desired size of the *Srichakra* to be drawn. It may be noted that construction of *Srichakra* requires the specification of only five points on the central vertical diameter AD. These are *five independent variables*. In the traditional method for constructing *Srichakra*, nine measurements are specified. These correspond to the vertical divisions on AD - the points marked Z, M, F, S, U, Y, C, K and X. Actually, all nine data points are not independent. The location of points M, S, U and K are derived during the construction procedure described above. The geometrical problem of *Srichakra* construction is then to determine the appropriate input data so that the closing error is ideally zero or practically very small.

3

Review of Existing and Proposed Data

In this Chapter we review available data on *Srichakra*. We also propose certain modifications to the existing data. We demonstrate that there exit multiple solutions (i.e., multiple sets of input data) for constructing *Srichakra*. We point out the need for a computational approach to study the geometry of *Srichakra*.

3. 1. Traditional Data

In the traditional data, (x, y) coordinates of the corners of all triangles are explicitly specified. Then the procedure of constructing *Srichakra* consists in merely joining the specified corners appropriately. See Fig.2.1 for the nomenclature. The vertical axis (AD) is divided into 48 equal parts. Each part is referred to as one unit. Column 1 of Table 3.1 shows the distance on the vertical axis from A to the horizontal base for the nine triangles. A chord is a straight line that has end points lying on the circumference of a circle. In *Srichakra*, there are only two such chords, viz., FE and CB. When the width of the horizontal base of a triangle is shorter than that of a chord, coinciding with that base, certain number of units on either side of the chord has to be erased. The number of units to be erased is given in Column 2 in Table 3.1. Given a point on the vertical diameter, one-half of the chord width can be computed using a formula (See Sec. 4.1.4). The computed base widths of the nine triangles are shown in Column 3 of Table 3.1 to two decimal places

Table 3.1.

Traditional Data to Construct *Srichakra*

Distance in units from the apex **A** (Distance relative to the centre at x=0, y=24).	The number of units to be erased on either side of the coinciding chord	Computed one-half of the base width = Computed one-half of chord width minus the data in Column 2
AZ = 06 (24-18)	3	ZZ1 = 15.87 - 3 = 12.87
AM = 12 (24-12)	**5**	MN1 = 20.78 - 5 = 15.78
AF = 17 (**24-07**)	0	FE = 22.96 - 0 = 22.96
AS = 20 (24-04)	16	ST = 23.66 - 16 = 7.66
AU = 23 (24-01)	18	UQ1 = 23.98 - 18 = 5.98
AY = 27 (24+03)	16	YP = 23.81 - 16 = 7.81
AC = 30 (**24+06**)	0	CB = 23.24 - 0 = 23.24
AK = 36 (24+12)	**4**	KL1 = 20.78 - 4 = 16.78
AX = 42 (24+18)	3	XX1 =15.87 - 3 = 12.87

The data given in Columns 1 and 2 of Table 3.1 constitute the traditional data and have been attributed to Sri Kaivalyashrama by some authors (Sri Subramanya Sastri and Sri Srinivasa Iyengar) and to Sri Lakshmidhara by others (Sri Ramalingeswar Rao, Sri MPS). It is said that Sri Kaivalyasrama and Sri Lakshmidhara have mentioned the above data in their commentary on *Saundarya Lahari* of Sri *Adi Shankaracharya*. It appears from the manner in which the data is specified that either Sri Kaivalyashrama or Sri Lakshmidhara has measured the various corner points of triangles from a traditional *Srichakra* that they had access to.

We came across a method in the work *Nityotsava* authored by Sri Umanandanatha, a disciple of Sri Bhaskararaya of the early 19th century. Sri Umanandanatha has described the constructional procedure in *Srushti krama*, which begins with the inner-most triangle (*yoni*) and proceeds outwards.. That description is qualitative and hence could not be validated.

Sri S. K. Ramachandra Rao mentions the following method in his book on *Srichakra*. Draw a circle of required diameter (AD). On the central vertical axis (diameter), draw nine *equi-distant* parallel horizontal lines to obtain the points, Z, M, F, S, U, Y, C, K, X. He then proceeds to describe a procedure for constructing *Srichakra*. However, this procedure gives rise to *Srichakra* with gross errors since the points on the vertical axis (diameter) are assumed to be *equi-distant*.

Aesthetic aspects of *Srichakra* drawing based on traditional data

The drawing of *Srichakra* obtained with the above data is shown in Fig.3.1a. The reason for showing some of the triangles of *Ashtakona* as filled will become apparent later in Sec. 3.7 when we discuss the trade-off between the aesthetic appeal *versus* the closing error.

On the vertical axis, the pair of points Z (y=24-18), X (y=24+18) and the pair of point M (24-12) and K (24+12) are symmetrically located with respect to the centre (y=24). The horizontal base through Z and X have the same width (12.87) giving rise to an aesthetic appeal. However, the horizontal base through M and K have a slight difference in widths (15.78, 16.78), since the number of units to be erased on either side of the horizontal chord through M and K are 5 and 4, respectively. On the vertical axis, the points F (24-7) and C (24+6) are almost symmetrically located with respect to the centre (differ only by 1 unit) and the horizontal base through F or C forms a chord giving rise to an aesthetic appeal. There are two points (S, U) above the centre whereas there is only one point Y below the centre giving rise to an asymmetry of four upward and five inverted triangles.

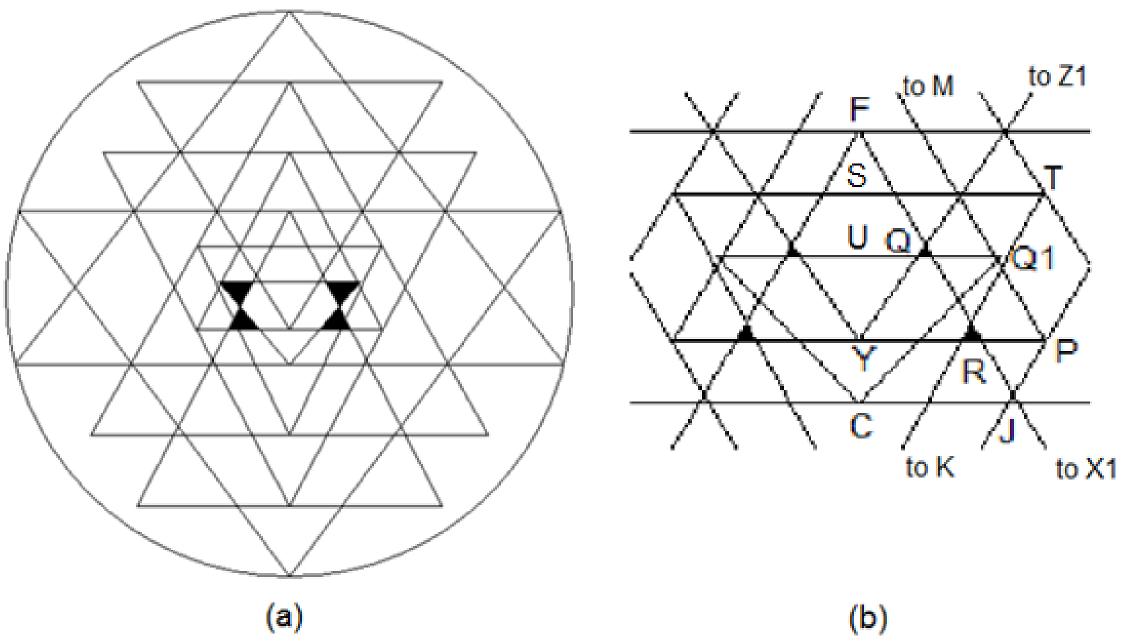


Fig. 3.1 (a) Srichakra drawn with traditional data of nine vertical and nine horizontal points as in Table 3.1. Some of the triangles in Ashtakona are shown filled but they don't represent any error. (b) An enlarged view of a part of Srichakra to visualize the gross errors (filled triangles)

Shat-chakra* and *Srichkra

Shat-chakra is another popular mystical drawing. In a *shat-chakra* the base of upward and inverted triangles form chords of a circle and the two chords are usually shown as equidistant from the centre. In some drawings of *sat-chakra*, the two triangles are shown as equilateral. One may be tempted to assume that *sat-chakra* is derived from *Srichakra* and that the two triangles of *shat-chkara* correspond to the triangles B'AB and E'DE. However, in *Srichakra*, the chords B'B and E'E are not equidistant as their mid-points C (24+6) and F (24-7) are not symmetrically located with respect to the centre (24). Subsequently, the widths (22.96, 23.24) of the base of the two triangles differ slightly (by 0.28 of a unit)..

Gross Errors in the drawing of *Srichakra* constructed based on traditional data

Despite the aesthetic aspects seen in Srichakra constructed with traditional data, there are some gross errors as described below. See the enlarged drawing shown in Fig.3.1b.

(a) The point Q1 goes beyond the line MP resulting in a gross error. This happens since Q1 has to satisfy two independent constraints: (i) the width of the horizontal base (UQ1) is derived based on the input data to be 5.98; (ii) The points M and P are also independently specified so as to draw the line MP. The horizontal from U is supposed to meet MP at Q1. In other words, MP is supposed to pass through Q1. This doesn't seem to happen as per the traditional data.

(b) All the six points, F, X1, Y, Z1, U and Q1 are independently specified as input data. The intersection point, Q, of the lines FX1 and YZ1 is supposed to lie on the line UQ1. Instead these two lines intersect slightly above the line UQ1. This error is shown by a filled triangle in Fig.3.1b.

(c) All the six points F, X1, Y, P, T and K are independently specified as input data. The lines FX1 and YP intersect at R. The line TK is supposed to pass through R. Instead, it passes slightly to the left of R. This error is shown by a filled triangle near R in Fig.3.1b.

3. 2. Kulaichev's Work

Mr. Alexey Pavlovich Kulaichev, a Russian scientist, has presented a technical paper in an Indian Journal in which he refers to *Srichakra* as *Sriyantra*. His motivation is claimed to be one of pure mathematical interest in the complexity of the drawing. He states (underlining ours) :

> "*The process of reproduction of Sriyantra has unexpectedly turned out to be a very difficult problem. Most of the seal's lines pass through 3-6 points of intersection of other lines, and a lot of redrawing of the whole figure is needed in order to attain precise super position of points*."

It may particularly be noted that Mr. Kulaichev refers to the process as requiring '*a lot of redrawing*'. Mr. Kulaichev also refers to an *old* method in which the circle is divided into 48 equal parts and horizontal lines are drawn through the 6th, 12th, 17th, 20th, 23rd, 27th, 30th, 36th and 42nd points (This is the same as the traditional data referred to above). He concludes that such an input data produces a visible inaccuracy, as we have already illustrated. He also notes that in some other drawings the point Z1 lies on or near the circumference of the circle.

He goes on to formulate the geometry of *Srichakra* as a mathematical problem and states that a huge amount of computational time is required to solve the same. He also believes that the original drawing might have been drawn on a sphere with arcs and that the plane *Srichakra* is a projection of the spherical drawing. He is of the opinion that such a drawing might have been handed down traditionally. The author seems to be overwhelmed by the complexity of the drawing as he states in his conclusion, that constriction of *Srichakra*

> "... *would involve a very high level of mathematical knowledge*."

> "*The Sriyantra as shown here is a very complicated and many sided object, and for its deep study it is required to apply efforts by specialists from different fields of knowledge: mathematics, history, ethnography, psychology, philosophy, etc.*"

The psychological effect of staring at *Srichakra* as compared to other geometrical figures has also been reported by Russian scientists Kulaichev and Ramendic. They report certain beneficial effects in some subjects.

3. 3. MPS's Method and Data

Sri M. P. Shankaranarayanan (MPS) has suggested a **method** for constructing *Srichakra* given only ***five*** input points along the vertical axis instead of nine input points as specified in the traditional data. Also, he ignores the data given on the widths of the horizontal lines given in the Table of Sec.3.1. Further, he has also suggested modification of the five input points of the traditional data to reduce the closing error.

MPS's Method

Referring to Fig. 2.1 on nomenclature, the five input data points relative to AD (= 48) are AZ=6, AF=17, AY=27, AC=30 and AX=42. We follow the steps given in Chapter 2.

The *Srichakra* obtained by following MPS's method of construction is shown in Fig.3.2a. Point Q1 doesn't extend beyond the sloping line MP as seen in Fig.3.1b. There are no gross errors despite the fact that the five data points used by MPS are the same as in the traditional data. The closing error (i.e., line TK not passing through R) may clearly be noted especially in the enlarged drawing (Fig.3.2b). The normalized closing error comes out to be 1.248%.

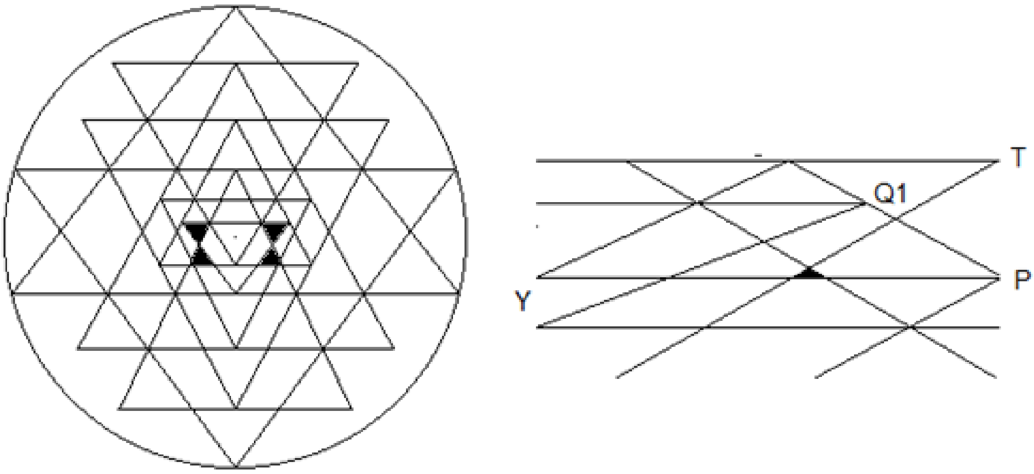
Srichakra drawing shown in Figs.3.1a is obtained using a set of nine vertical and nine horizontal data points whereas *Srichakra* shown in Fig. 3.2a is obtained using only five input data on the vertical axis. The two drawings are shown superposed in Fig.3.3a. Significant differences are seen in the height of horizontal base UQ1 (fig.3.3b) and consequently in the inclined line CQ1. Also, slight difference arises in the width of XX1 and in the locations of M and K.

MPS's suggested modification of the traditional data

MPS suggests an improvement over the traditional data in order to reduce the closing error. He uses 100 units for AD instead of 48 units. Then the five of the traditional input data points on the vertical axis are scaled-up proportionately to yield AZ=(6/48)X100=12.55, AF=(17/48)X100 = 35.42, AY=(27/48)X100 = 56.25, AC=(30/48)X100 = 62.5 and finally AX=(42/48)X100= 87.5. MPS further modified these to 12.05, 36.20, 55.48, 62.87 and 87.70 units, respectively. MPS has not given the rationale based on which he has modified the scaled-up data. They seem to have been obtained by trial and error.

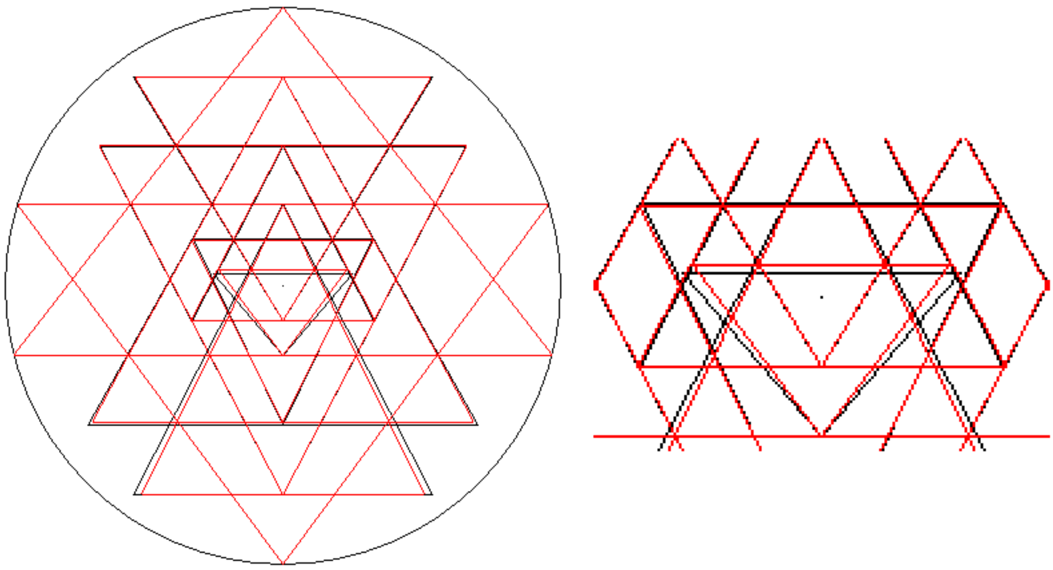
In order to facilitate easy remembrance of the odd fractions, MPS has constructed a poem in *Anushtup* and coded the fractions using the well known *Katapayadi Sutra*. Just because the poem is in Sanskrit, it should not be mistaken to be a quotation from an authentic traditional source.

With the input data proposed by MPS, a more accurate drawing is obtained as shown in Fig.3.4a. Now, the normalized closing error is 0.1135% instead of 1.248%. That is, the closing error is reduced by a factor of about ten. This error is not noticeable to the naked eye. It cannot be recognized even in an enlarged drawing (Fig.3.4b). However, *theoretically* it can be argued that MPS's data is still inaccurate because of finite closing error.



(a) (b)

Fig. 3.2 (a) Srichakra with only five input points selected out of traditional data as proposed by MPS. Some of the triangles in Ashtakona are shown filled but they don't represent any error. (b) An enlarged view of a part of Srichakra to visualize the closing error shown by a filled triangle.



(a) (b)

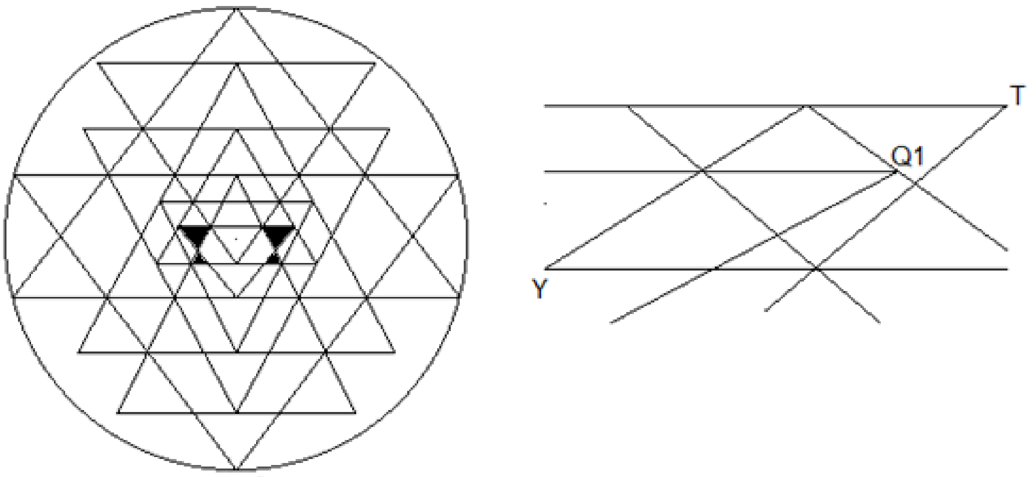
Fig. 3.3 (a) Superposed Srichakra drawings, one (in Black) constructed using the traditional nine vertical and nine horizontal data points and the other (in Red) constructed with only five vertical input points (b) An enlarged view of a part of Srichakra to visualize the differences.

Table 3.2.

Traditional Data, Scaled-up Traditional Data,

MPS's Suggested Modification and Data proposed by TVA and MNR

	Traditional Data for		MPS's Data	TVA & MNR Data
	AD=48	AD=100	AD=100	AD=108
AZ	6	12.55	12.05	13
AF	17	35.42	36.20	39
AY	27	56.25	55.48	60
AC	30	62.50	62.87	68
AX	42	87.50	87.70	94



(a) (b)

Fig. 3. 4. (a) Srichakra constructed using MPS's suggested data. Some of the triangles in Ashtakona are shown filled but they don't represent any error. (b) An enlarged view of a part of Srichakra It is difficult to notice any closing error.

3. 4. Traditional Data Vs MPS's Data

Traditional data are extremely simple to implement even by a layman since integer divisions are to be marked along the diameter and integer parts are to be erased from the chords. A good choice is to use a scale graduated in (a) eighths of an inch and assume a diameter of 6 inches or (b) sixteenths of an inch and assume a diameter of 6 inches or (c) tenths of an inch for a diameter of 4.8 inches or (d) assume diameter of 96 mm (9.6 cm). The resolution in these cases, viz., one unit relative to the diameter is, respectively, (a) eighths of an inch or (b) two-sixteenths of an inch or (c) tenths of an inch (d) 2 mm.

Manual implementation of MPS's data is impractical. Let us say that we want to .construct a *Srichakra* of 100 mm (10 cm) diameter. In order to mark the input points on the vertical axis, a resolution of one hundredth of a milli-meter (0.01 mm).is required, which is impractical.

In fact, slight deviations in the marking may result in a larger closing error. Thus, if the data given by MPS is not correctly marked, a larger closing error might result in the drawing (See Sec.3.6 and Sec.5.3). However, for the sole purpose of calculating only the closing error using a computer program, one may make use of input data to any desired resolution.

MPS has made many useful contributions. He describes a systematic procedure for constructing *Srichakra* given only *five input data points* which we have adopted in Ch.2. More importantly, he derives the relevant coordinates in a sequence of steps and calculates the closing error given the input data (See Chapter 4). This is extremely useful as the closing error can be calculated without actually constructing *Srichakra* and thus the validity of the input data can be determined. If the closing error is large, then the input data can be rejected outright as invalid.

3. 5. Data Proposed by the Authors (TVA and MNR)

We wondered if it is not possible to combine the merits of traditional data of integer divisions for easy marking with the more accurate data, similar to that suggested by MPS, resulting in a low closing error.

We speculate that the choice of 100 units for the diameter as proposed by MPS as arbitrary. Instead, we select AD as 108 units since it is a mystical number close to 100. The choice of 108 units is also prompted by other considerations. The simplest form of *Srichakra*, *sat-chakra*, resembles a star. *Srichakra* is supposed to represent the space and time of cosmos. This once again is related to the movement of stars. There are twenty-seven stars, with each star having four *padas* (quarters), resulting in a totla of 108 *padas* (quarters) in the Zodiac. *Srichakra* is called *nava-avarana chakra* and 108 (9X12) happens to be an integer multiple of 9. We have already noted in Chapter-1 that there are 108 *Bhunavas* in *Prithvi tatva*.

Based on these observations, we consider the diameter AD as 108 units. Then, scaling-up MPS's data (Column 3 of Table 3.2) by a factor of 108/100, results in the data 13.01, 39.09, 59.91, 67.899 and 94.716; Since we desire to have the divisions as integers *a la* the traditional method, these values have been changed to the nearest integers as 13, 39, 60, 68 and 94 (not 95) units (Column 4, Table 3.2). *Srichakra* obtained with the proposed data is shown in Fig.3.5 and it results in a normalized closing error of 0.12%. This closing error is one-tenth of that obtained using the traditional data and is close to that obtained using MPS's data (0.1135%). The proposed data has the practical advantage of easy (integer division) marking. See Sec. 3.6 below for further improvement in the proposed data.

It is easy to remember the desired markings by referring the divisions to *padas* of the stars as follows. For example, the integer divisions at 13, 39, 60, 68 and 94 units correspond to *Rohini* 1st *pada*, *Makha* 3rd *pada*, *Swathi* 4th *pada*, *Anuradha* 4th *pada* and *Shatabhisa* 2nd *pada* respectively.

The advantage of the proposed data is that one can draw a circle of 108 mm and mark the divisions of input data to an accuracy of 1 mm. This is a relatively an easier task compared to the resolution of 0.01 mm required using MPS's data. Alternately, one can choose a diameter of 9 inch with one unit as one-twelfths of an inch or a diameter of 10.8 inch with one unit as one-tenths of an inch.

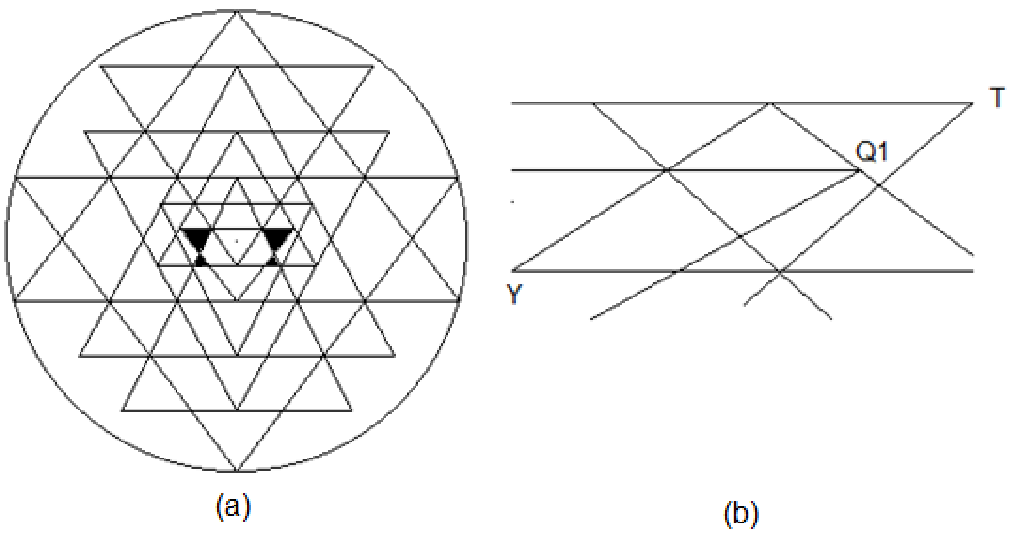


Fig. 3.5 (a) Srichakra drawn with data proposed by the authors (TVA and MNR). Some of the triangles in Ashtakona are shown filled but they don't represent any error. (b) An enlarged view of a part of Srichakra It is difficult to notice any closing error.

3. 6. Sensitivity of the Closing Error to the Input Data Specification

We would like to introduce an important concept called sensitivity. In converting MPS's data to the proposed data, we converted AX=94.716 to integer value of 94 instead of 95. When AX=95 is used, the normalized closing error comes out to be larger (0.22%), which is nearly twice the normalized closing error (0.12%) obtained for the case of AX=94. This prompted us to try AX=94.5. This choice gave a normalized closing error of 0.05% which is about one-half of the error compared to the case of AX=94. *Srichakra* obtained for the case of AX=94.5 is shown in Fig.3.6. It is not impractical to mark up to an accuracy of 0.5 mm. When the closing error is less than the line thickness, the error will not be visible.

The normalized closing errors for AX=94, 94.5 and 95 are respectively 0.22%, 0.05%, 0.12%. This result implies that the closing error is highly sensitive to the value of AX. Closing error is sensitive not only to the value of AX but also to other input data points such as AZ, AF, AY and AC. The highly sensitive nature of closing error and the non-availability of accurate methods might have prompted the use of integer divisions in the olden times. See Sec.5.3 for a further discussion on the sensitivity.

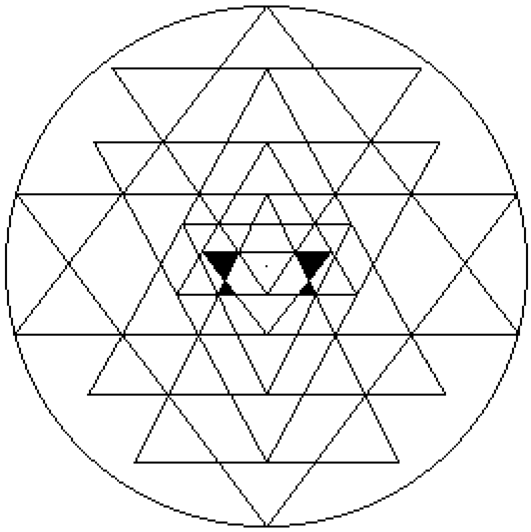


Fig.3.6. Srichakra drawn with proposed data and with a resolution of 0.5 mm. Some of the triangles in Ashtakona are shown filled but they don't represent any error.

3. 7. Trade-off Between Aesthetic Appeal and the Closing Error

Compare *Srichakra* shown in Figs.3.1 to 3.6. In particular, note the location of the point Z1. As we try to improve upon the traditional data to reduce the closing error, the point Z1 moves closer to the circumference of the circle as compared to *Srichakra* drawn using the traditional data (Figs.3.1 and 3.2). The widths of the base of upper-most (Z1'Z1) and lower-most (X1'X1) triangles are exactly equal and the areas of the filled triangles of *Ashtakona* are nearly equal in *Srichakra* constructed with the traditional data (Figs.3.1 and 3.2). On the other hand, in *Srichakra* illustrated in Figs. 3.3 to 3.6, the widths Z1'Z1 and X1'X1 are significantly different and the filled triangles of Ashtakona appear are highly disproportionate. This seems to suggest a trade-off between aesthetic appeal and the closing error.

We get a very low closing error of 0.0005% for input 13.5, 40.0, 59.5, 68 and 96 units (AD=108 units). This error may be considered to be practically zero. *Srichakra* obtained for the above choice of input data is shown in Fig.3.7a. Although the closing error is extremely small (practically zero) other undesirable features may be noted. The point Z1 lies almost on the circumference of the circle. Line TK almost touches point Q1 making some of the triangles in *Ashtakona* almost vanish (see the filled trianlges). The above inference should not be generalized. It is possible to obtain a drawing with a very small closing error and yet with the point Z1 well within the circle. Such a *Srichakra* obtained for input data AZ=13.5, AF=38, AY=60, AC=68.5, AX=94 is shown in Fig.3.7b. The closing error for this data is 0.000327%. It may be seen that the point Z1 lies well within the circle. However, some of the triangles of *Ashtakona* almost vanish (see the filled triangles).

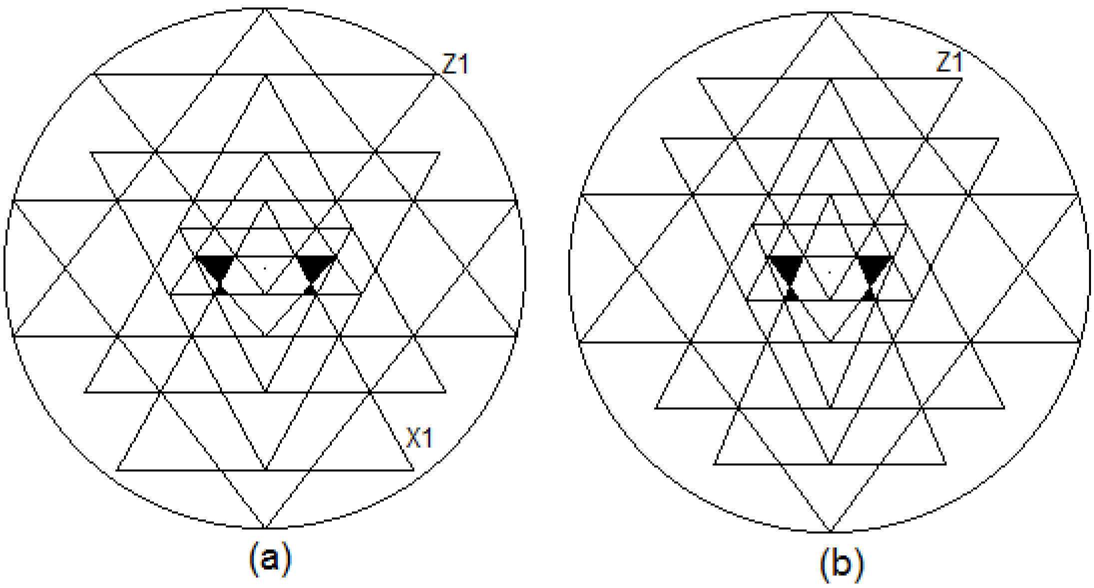


*Fig.3.7 (a) An example of Srichakra with a very closing error (*0.0005%) but with (Z1, X1) almost lying *on the circumference of the inner-most circle* (b) *An example of Srichakra where the point Z1 is interior and farther from the circumference yet with a very closing error of* 0.000327%*..*

3. 8. Other Errors

Judgment based only on the closing error could be misleading. If the input data (five points relative to AD) are improperly specified then errors other than closing error are encountered. Even when the input data are accurately given but are not correctly marked due to practical limitations, certain other errors occur. Some such errors other than the closing error are discussed below.

3. 8. 1. Base Error: Sometimes the point Z1 touches or goes beyond the circumference of the circle (Fig.3.7a). Similarly, errors are encountered when the point X1 goes beyond the circumference of the circle although such cases are rare.

3. 8. 2. *Ashtakona* Error : When the point Q2 lies to the left of Q1 (See Fig.2.2 on the nomenclature), it is a definite case of *Ashtakona* error. When the line TK passes close to W then the area of *Ashtakona* triangle near R, triangle VWR (refer Fig.2.2) becomes negligible and may almost vanish. From an aesthetic point of view the areas of the filled triangles, VWR and VQQ1 should be nearly equal.

From the above discussion it follows that the closing error alone is an insufficient criterion to assure us of an aesthetically appealing non-erroneous drawing of *Srichakra*. All aspects of *Srichakra* have to be taken into account. See also Sec. 4.4.3 to 4.4.6 for other error criteria.

3. 9. Need for a Computational Method

We have seen several drawings of *Srichakra* for different sets of input data implying that there are multiple solutions. It follows that one may find a large number of solutions which satisfy the desired criteria of acceptability. Then one would have to select from these valid or acceptable solutions, a smaller set based on additional criteria such as aesthetic appeal.

From a *theoretical point of view* it is not necessary to restrict the input data to be integers as a high degree of accuracy can be achieved using computational methods. If we ignore the practical difficulty of hand marking the divisions and manually constructing *Srichakra*, then the problem is one of searching for all acceptable solutions. Using a computer program, a large number of choices for input data of any desired resolution for various error criteria can be tested without actually constructing *Srichakra*. If the errors are below acceptable limits and the drawing is aesthetically appealing then the input data may be assumed as valid and the corresponding *Srichakra* may be constructed or plotted for visualization. Computational method is discussed in the next Chapter. Results obtained using the computational method are presented in Chapter 5.

4

Computational Method

This Chapter is highly technical. It requires a basic knowledge of geometry to understand the algorithm and a basic knowledge of computer programming to understand the implementation of the algorithm on a computer. Readers not interested in technical details may skip this Chapter and directly go to the next Chapter. Even non-technical readers can appreciate the results of the computational method presented in the next Chapter.

4. 1. Basic Geometrical Concepts

In the construction of *Srichakra* some basic geometrical concepts are used. These concepts are presented in this Section.

4. 1. 1. Coordinate system

We use a rectangular coordinate system. The origin is defined as the centre of the inner-most circle. The y-axis is represented by the central vertical diameter of *Srichakra*. The x-axis is represented by a horizontal line passing through the centre of the inner-most circle. See Fig.4.1. With this choice, the y-coordinate has negative values for points below the centre. The x-coordinate has negative values for points on the left half of the inner-most circle.

4. 1. 2. Equation of a straight line

A straight line is given by the general equation of the form

$$y = \mathbf{m}x + \mathbf{c}$$

where **m** is the slope of the line and **c** is the intercept on the y-axis (Fig.4.2). The slope is negative if the line starts from top left hand side and goes towards bottom right hand side. On the other hand the slope is positive when the line goes from bottom left hand side to the top right hand side. A horizontal line has a constant y-coordinate. A vertical line has a constant x-coordinate.

Given the coordinates (x1, y1) and (x2, y2) of two points on a line, the slope **m** can be calculated using the formula (Fig.4.2.)

$$\mathbf{m} = (y2 - y1) / (x2 - x1) \tag{4.1}$$

The intercept on the y-axis **c** is obtained using either of the equations: y1=**m**x1+ **c** or y2=**m**x2 + **c**.

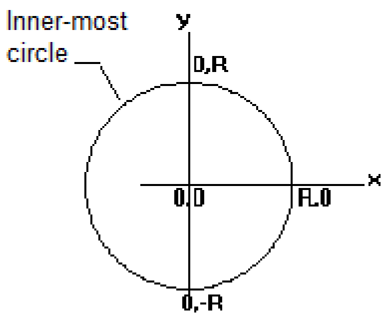


Fig. 4.1

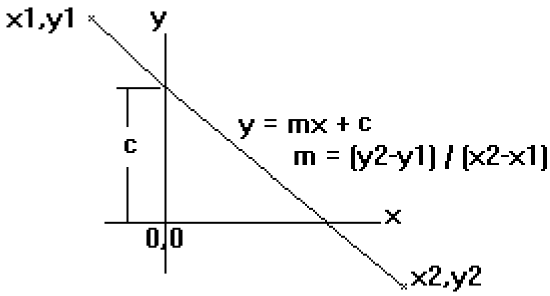


Fig. 4.2

4. 1. 3. Point of intersection of two lines

Let y = **m1**x + **c1** and y = **m2**x + **c2** be two straight lines intersecting at (x0, y0). (See Fig.4.3.) Then the coordinates x0 and y0 are given by

x0 = **(m2 - m1) / (c2 - c1)** and (4.2a)

y0 = **m1**x0 + **c1 or m2**x0 **+ c2** (4.2b)

4. 1. 4. Chord width

A straight line joining two points on the circumference of a circle is called a chord (Fig.4.4). Perpendicular bisector of a chord is a diameter of the circle. That is the perpendicular bisector passes through the centre of the circle. Point of intersection of the chord and the bisector divides the diameter into two parts. There is an interesting mathematical relationship between these two parts and the chord width.

In Fig.4.4, half the chord width FE is related to the two parts on the diameter AD by the formula

$$FE^2 = (AF)(FD) = (AF)(AD-AF) \quad (4.3)$$

Given AF and AD, half the chord width FE can be calculated.

4. 1. 5. Similar triangles

Fig.4.5 shows two similar triangles. The sides of the similar triangles satisfy the following relation

$$ZZ1/ZY = FI/FY \text{ or } ZY/FY = ZZ1/FI$$

$$ZZ1 = ZY\,[FI/FY] \text{ or } ZY = ZZ1\,[FY/FI] \quad (4.4)$$

Thus given FI, FY and ZY, then ZZ1 can be calculated.

An alternative method of writing the above result is using trigonometric relation as follows:

$\tan(FYI)=FI/FY = \tan(ZYZ1) = ZZ1/ZY$ or $ZZ1= [\tan(FYI)]\, ZZ1$ or $ZY= ZZ1 \cot(FYI)$

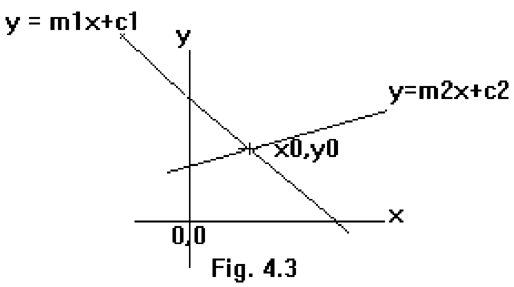


Fig. 4.3

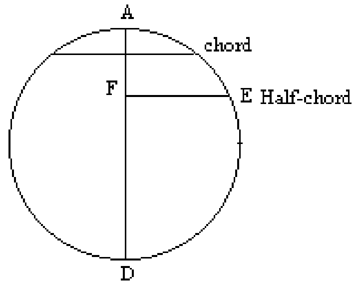


Fig. 4.4

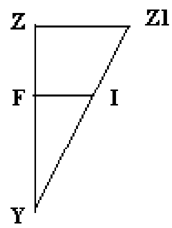


Fig. 4.5

4. 2. Algorithm

For a given set of input data [AZ, AF, AY, AC, AX and AD], the coordinates of all the relevant points of *Srichakra* are computed. Then the errors are computed to verify if *Srichakra* satisfies the desired criteria. The algorithm is described in the following steps. These steps closely match the steps given in Sec.2.2. Refer to Fig.2.3.

(1) Radius of the circle R = AD/2 .

(2) The coordinates of the input data are determined. Note that the input data lie on the y-axis and hence x-coordinate of all the input data is zero.

x(A) = 0 , y(A) = R.	X(D) = 0, y(D) = -R.
x(Z) = 0, y(Z) = R - AZ.	x(F) = 0, y(F) = R - AF
x(Y) = 0, y(Y) = R - AY	x(C) = 0, y(C) = R - AC.
x(X) = 0, y(X) = R - AX.	

Since AY, AC and AX are greater than R, y(Y), y(C), y(X) have negative values.

(3) Find the coordinates of E. Refer Eq.(4.3a).

x(E) = Half Chord Width FE, where
FE^2 = (AF)(FD) = (AF)(AD-AF)
y(E) = y(F) (Given)

Equation for FE is given by y = y(F). Note that the slope of line, FE, **m**(FE) is zero. The intercept on the y-axis **c**(FE)=y(F).

(4) Find the coordinates of B. Refer Eq.(4.3a).

x(B) = Half Chord Width CB where
CB^2 = (AC)(CD) = (AC)(AD-AC)
y(B) = y(C) (Given)

Equation for CB is given by y = y(C). Note that the slope of line CB, **m**(CB) is zero. The intercept on the y-axis is **c**(CB)=y(C).

(5a) Determine the equation of the line AB. The coordinates of A and B are known. Use Eq. 4.1 to determine the slope.

m(AB) = [y(A) - y(B)] / [x(A) - x(B)]

Intercept on the y-axis is **c**(AB) = R.

(5b) Find point G. G is the intersection of AB and FE. Use Equations (4.2a) and (4.2b)..

m1 = m(AB), **c1** = y(A); **m2**= 0, **c2** = y(F).

Find x(G) and y(G).

(6a) Determine the equation of the line DE since the coordinates of D and E are known. Use Eq. 4.1 to determine the slope.

m(DE) = [y(D) - y(E)] / [x(D) - x(E)]

Intercept on the y-axis is **c**(D) = -R.

(6b) Determine the point H. H is the point of intersection of CB and DE. Use Equations (4.2a) and (4.2b).

m1 = **m**(CB) = 0, **c1** = y(C)

m2= **m**(DE), **c2** = y(D)

Find x(H) and y(H).

(7a) Determine the equation of the line ZH.

m(ZH) = [y(Z) - y(H)] / [x(Z) - x(H)]

c(ZH**)** = y(Z).

(7b) Determine coordinates of I. I is the point of intersection of ZH and FE. Use equations (4.2a) and (4.2b).

m1 = **m**(ZH), **c1**= y(Z).

m2 = 0, **c2** = y(F).

Find x(I) and y(I).

(8) Find the equation of line YI.

m(YI) = [y(Y) - y(I)] / [x(Y) - x(I)]

c(YI) = y(Y)

(9) Find ZZ1 from similar triangles YIF and YZ1Z.

ZZ1 / ZY = FI / FY

ZZ1 = ZY [FI/FY]

x(Z1) = ZZ1 = [y(Z) - y(Y)] [x(I) / { y(F) - y(Y) }]

y(Z1) = y(Z)

(10) N is the point of Intersection of AB and YZ1.

m1 = **m**(AB), **c1** = y(A).

m2 = **m**(YZ1), **c2** = y(Y).

Find x(N) and y(N).

(11) Find the point J. J is the intersection of XG and CB.

m1 = **m**(XG) = [y(X) - y(G)] / (x(X) - x(G)]

c1 = y(X).

m2 = **m**(CB) = 0. **c2** = y(C)

Find x(J), y(J)

(12) Find M. y(M) = y(N). x(M) = 0.

(13) Produce XG to N1. That is, find the width MN1 and hence the coordinates of N1. From similar triangles, XMN1 and XFG,

MN1/ MX = FG / FX

MN1 = MX [FG / FX]

x(N1) = MN1 = [y(M) - y(X)] [x(G) / { y(F) - y(X) }]

y(N1) = y(N) = y(M)

(14) Produce FJ to X1. That is, find the width XX1 and hence the coordinates of X1. From similar triangles, XFX1 and CFJ

XX1 / FX = CJ / FC

XX1 = FX [CJ / FC]

XX1 = x(X1) = [y(F) - y(X)] [x(J) / { y(F) - y(C) }]

y(X1) = y(X)

(15) Find L. L is the point of intersection of FX1 and DE.

m1 = **m**(FX1) = [y(F)- y(X1)] / [x(F) - x(X1)], **c1** = **c**(FX1) =y(F)

m2 = **m**(DE), **c2** = y(D)

Find x(L), y(L).

(16) Find Q. Q is the point of intersection of FJ and YN

m1 = m(FJ) = [y(F)- y(J)] / [x(F) - x(J)], **c1** = **c**(FJ) = y(F)

m2 = **m**(YN) = [y(Y) - y(N) / [x(Y) - x(N)], **c2** = y(Y).

Find x(Q), y(Q).

(17) Find K. y(K) = y(L).   x(K) = 0.

(18) Produce ZH to intersect KL produced, at L1. That is, find the width KL1 and hence coordinates of L1. From similar triangles ZKL1 and ZCH,

KL1 / ZK = CH / ZC

KL1 = [ZK] [CH/ZC]

x(L1) = KL1 = [y(Z) - y(K)] [x(H) / {y(Z) - y(C)}]

y(L1) = y(K)

(19) Horizontal through Y meets FX1 at R. From similar triangles FYR and FXX1,

YR / FY = XX1 / FX

YR = [FY] [XX1 / FX]

x(R) = YR = [y(F) - y(Y)] [xX1 /{ y(F) - y(X) }]

y(R) = y(Y).

(20) Horizontal through Y meets XG at P. From similar triangles, YXP and FXG

YP / YX = FG / FX

YP = [YX] [FG / FX]

x(P) = YP = [y(Y) - y(X)] [x(G) / { y(F) - y(X) }]

y(P) = y(Y)

(21) T1 is the point of intersection of MP and YZ1.

m1 = **m**(MP) = [y(M) - y(P)] / [x(M) - x(P)], **c1** = **c**(MP) = y(M)

m2 = **m**(YZ1) = **m**(YI). **c2** = y(Y),

Find x(T1) and y(T1).

(22) Horizontal through Q meets AD at U.

y(U) = y(Q). x(U) = 0.

(23) Produce UQ to Q1. That is, find width UQ1 and hence the coordinates Q1. From similar triangles MUQ1 and MYP,

UQ1 / MU = YP / MY

UQ1 = [MU] [YP / MY]

$x(Q1) = UQ1 = [\ y(M) - y(U)\ ]\ [\ x(P) / \{\ y(M) - y(Y)\ \}]$

$y(Q1) = y(Q)$.

(24) Find the equation of CQ1.

$\mathbf{m}(CQ1) = \{y(C) - y(Q1)\}/\{x(C) - x(Q1)\}$: $\mathbf{c}(CQ1) = y(C)$

(25) Horizontal through T1 meets AD at S.

$y(S) = y(T1)$. $x(S) = 0$.

(26) Produce ST1 to T. That is, find width ST and hence coordinates of T. From similar triangles ZST and ZCH,

$ST / ZS = CH / ZC$

$ST = [\ ZS]\ [\ CH / ZC\ ]$

$x(T) = ST = [\ y(Z) - y(S)\ ]\ [\ x(H) / \{\ y(Z) - y(C)\ \}]$, $y(T) = y(T1)$.

(27) Find the equation of TK.

$\mathbf{m}(TK) = \{y(T) - y(K)\} / \{x(T) - x(K)\}$, $\mathbf{c}(TK) = y(K)$

(28) Closing Error

Let TK intersect FJ at R1 (See Fig.2.2). Find the coordinates of R1.

$\mathbf{m1} = m(TK)$, $\mathbf{c1} = y(K)$

$\mathbf{m2} = m(FJ)$ $\mathbf{c2} = y(F)$

Find $x(R1)$, $y(R1)$

Distance between R and R1, D(R,R1), is calculated using the formula:

$[D(R,R1)]^2 = [\ x(R) - x(R1)\ ]^2 + [\ y(R) - y(R1)\ ]^2$

Closing Error = D(R,R1)

Normalized Closing Error % = 100 [D(R,R1)/ AD]

4. 3. Computer Program

The listing of a computer program written in QuickBasic Version 4.0 (also runs on QBASIC) implementing the above algorithm is given in Appendix-A. The program is written assuming that a VGA colour monitor is used. Appropriate changes may be made to suit other monitors. Latest implementation has been made using Microsoft's Visual Basic ver.5. The steps in the computer program closely match the algorithm given above and the construction described in Sec.2.2. Comments are given so that a person with a basic knowledge in programming can easily follow the logic when read along with the algorithm described above.

4. 4. Methodology

Construction of *Srichakra* from a geometrical point of view has been studied by varying the input data and computing the error criteria. The choice of input data, resolution and error criteria used in obtaining the results are presented in this Section. In all of the computational experiments AD has been chosen to be 108 units.

4. 4. 1. Range

It may be recalled that a very low normalized closing error (0.05%) without other types of errors has been obtained for the following choice of input data: AZ=13, AF=39, AY=60, AC=68 and AX=94.5. The input data is varied over a range of $\pm$2 units around these as the mean values as given below:

Input Data	Range
AZ	11 to 15
AF	37 to 41
AY	58 to 62
AC	66 to 70
AX	92.5 to 96.5

4. 4. 2. Resolution

In most of the experiments each of the input data is varied in steps of 0.5 unit. For example, the choice of 0.5 means that the following cases of AZ have been tried: 11, 11.5, 12, 12.5, 13, 13.5, 14, 14.5 and 15 (totally 9 cases). Similarly other variables (AF, AY etc) are also varied in steps of 0.5. This results in a total of 59049 trials.

In some experiments, smaller resolutions of 0.05, 0.01 and 0.005 have also been used. In such cases the range would be narrowed to one of the acceptable solutions found for a higher resolution. For example, let us say that one of the acceptable solutions is for AX = 11.5. Then the range for AX would be 11.6 to 11.7 but resolution (step size) would be either 0.05 or 0.01 or 0.005.

Theoretically, one can set the limit for the closing error very close to zero. Search for an ideal solution requires a huge amount of computational time and perhaps may call for an infinite resolution in the specification of input data. We have adopted a more pragmatic approach. When the normalized closing error is less than 0.05%, the error at (*Marma*) R is not noticeable even after magnification. Hence, cases with closing error less than 0.05% are assumed to be *practically* valid drawings. Further, when the closing error is less than the line thickness the error is not noticeable.

4. 4. 4. Base Errors and *Ashtakona* Error

In the example shown in Fig.3.8 we noted that although the closing error is very small the point Z1 lies almost on the circumference of the circle. During a search for solutions of *Srichakra* using a computer program a low closing error may be found but the horizontal lines through Z, M, K and X may cross the circumference of the circle. In such cases the solutions are invalid. We refer to such cases as base errors.

Computing base errors

(i) Let ZZ1 produced intersect the circle such that the chord width from Z to the circumference is Z-chord. Then

$$[\text{Z-chord}]^2 = (AZ)(AD-AZ)$$

ZZ1 must be less than Z-chord else there is a base error at Z.

(ii) Let MN1 produced intersect the circle such that the chord width from M to the circumference of the circle is M-chord. Then

$$[\text{M-chord}]^2 = (AM)(AD-AM)$$

MN1 must be less than M-chord else there is a base error at M.

(iii) Let KL1 produced intersect the circle such that the chord width from K to the circumference of the circle is K-chord. Then

$$[\text{K-chord}^2 = (AK)(AD-AK)$$

KL1 must be less than K-chord else there is a base error at K.

(iv) Let XX1 produced intersect the circle such that the chord width from X to the circumference of the circle is X-chord. Then

$$[\text{X-chord}]^2 = (AX)(AD-AX)$$

XX1 must be less than X-chord else there is a base error at X.

Computing *Ashtakona* error

Let TK intersect UQ produced at Q2 (see Fig. 2.2). Then Q2 should lie to the right of Q1. That is x(Q2) must be greater than x(Q1). Whenever x(Q2) is greater than x(Q1) the solution is assumed to be valid else it is a case of *Ashtakona* error.

4. 4. 5. Optional symmetry criterion 1 for *Chatur-dashara chakra*

This is strictly not an error but a constraint. Subjectively we have arrived at a criterion that produces an aesthetically appealing drawing. Refer to Fig.2.1 for the nomenclature.

(i) Let ZZ1 intersect AB at Z2.. Then the area A(Z) of the triangle Z1NZ2 and Area A(N) of triangle N1GN are given by

Area of the triangle = Half base times Altitude

$$\text{Area }(Z) = 0.5 * [x(Z1) - x(Z2)]\,[y(Z) - y(N)]$$

$$\text{Area }(N) = .5 * [x(N1) - x(N)]\,[y(N) - y(G)].$$

We compute the ratio of the difference to the average of the above areas as follows :

asym1 = [Area(Z) - Area(N)] / [0.5Area(Z) + 0.5Area(N)]

If absolute value of asym1 is less than 0.4 (i.e., 40%) then (subjectively) the figure is considered to be nearly symmetrical. By absolute value of asym1 is meant that the if asym1 is negative then the sign is ignored.

(ii) Let XX1 intersect DE at X2.. Then the area A(X) of the triangle X1LX2 and Area(L) of the triangle L1HL are given by

Area (X) = .5 * [x(X1) - x(X2)] [y(L) - y(X)]
Area(L) = .5 * [x(L1) - x(L)] [y(H) - y(L)].

We compute the ratio of the difference to the average of the above areas as follows :

asym2 = [Area(X) - Area(L)] / [0.5Area(X) + 0.5Area(L)]

If absolute value of asym2 is less than 0.4 (i.e., 40%) then (subjectively) the figure is considered to be nearly symmetrical.

4.4.6 Optional symmetry criterion 2 for *Ashtakona chakra*

This is strictly not an error but a constraint. Refer to Fig. 2.2 for Nomenclature. The area A(Q) of the triangles VQ1Q and area A(V) of triangle VWR are calculated. The desired coordinates of V and W are derived. We compute the ratio of the difference to the average of the above areas as follows:

asym8 = [Area(Q) - Area(R)] / [0.5Area(Q) + 0.5Area(R)]

If the absolute value of asym8 is less than 0.4 (i.e., 40%) then (subjectively) the figure is considered to be nearly symmetrical.

5

Results of Computational Method

Results of computational method are presented in this Chapter.

5. 1. Multiple Solutions

This experiment is conducted to see if zero or near zero closing error can be achieved. For this purpose a resolution of 0.01 unit is used. Also, cases with base errors and *Ashtakona* error are excluded. The optional symmetry criterion for *Chatur-dashara* and *Ashtakona* are ignored. A large number of theoretically valid solutions have been obtained. Some selected examples are shown in Fig. 5.1.

For the choice of input data [13.35, 39.40, 59.55, 67.00, 93.20 and 108] the closing error is extremely small 0.0000076%. The point Z1 is very near the circumference of the circle. The points Q1 and Q2 are very close. (See Fig.5.1a).

For the choice of input data [12.45, 38.95, 60.25, 67.55, 93.35 and 108] the closing error is very small 0.000025%. The point Z1 is very near the circumference of the circle but not outside the circle. The points Q1 and Q2 are not very close giving a good *Ashtakona*. (See Fig.5.1b).

For the choice of the input data [13.45, 38.0, 59.95, 68.1, 93.4 and 108] the closing error is 0.000048%. This gives nearly a symmetrical shape for the *Chatur-dashara* but Q1 and Q2 are very close. (See Fig.5.1c).

For the choice of input data [13.80, 38.10, 60.00, 68.10, 93.10 and 108] the closing error is small 0.00005%. This gives nearly a symmetrical shape for the *Chatur-dashara* but Q1 and Q2 are very close. (See Fig.5.1d).

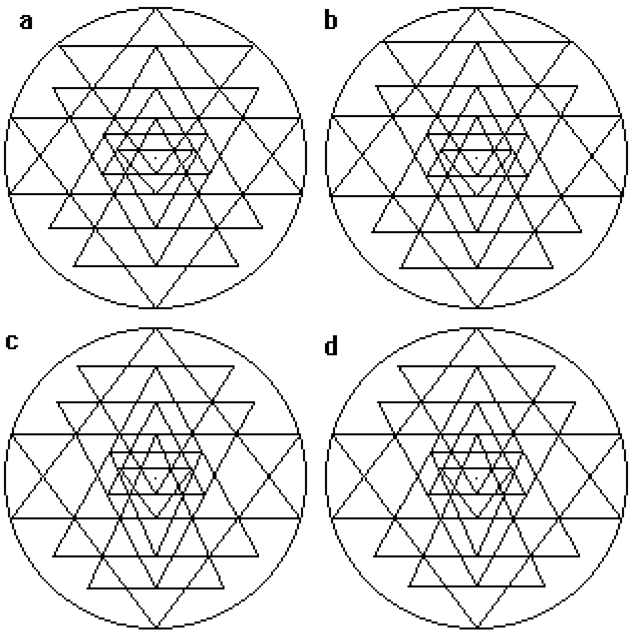


Fig.5.1. Some select Srichakra drawings having very low closing error:
(a) 0.0000076% (b) 0.000025% (c)0.000048% and (d) 0.00005%.

5. 2. Practical and Aesthetic Solutions

To limit the solutions to practically implementable cases, the resolution is restricted to 0.5 unit. The whole range as given in Sec. 4.4.1 is used. With this choice, there are 59049 trials. We select solutions where the closing error is less than 0.05% with no 'Base Errors'. When cases with *Ashtakona* error (i.e., error if xQ2 less than xQ1) are excluded but not the symmetry criterion for *Chatur-dashara*, 826 valid solutions are obtained. Of these, twenty two solutions are aesthetically appealing drawings whose input data and closing error are given in the Table 5.1. The drawings of *Srichakra* corresponding to these solutions are shown in Fig.5.2.

Table 5. 1.

Twenty Two Valid and Aesthetic Solutions of *Srichakra*

(Resolution = 0.5 Unit, AD=108 Units)

Sl No.	AZ	AF	AY	AC	AX	Closing Error %
1	11.0	37.5	61.5	70.0	96.0	0.017055
2	11.5	37.0	60.0	68.5	95.0	0.029634
3	11.5	37.5	61.0	69.5	95.5	0.013536
4	12.0	37.0	59.5	68.0	94.5	0.034498
5	12.0	37.5	60.5	69.0	95.0	0.013589
6	12.0	38.0	61.0	69.5	95.5	0.025982
7	12.5	37.0	59.0	67.0	93.0	0.021944
8	12.5	37.0	59.0	67.5	94.0	0.042469
9	12.5	37.0	59.5	68.0	94.0	0.016645
10	12.5	37.5	60.0	68.5	94.5	0.017023
11	12.5	38.0	60.5	69.0	95.0	0.022033
12	12.5	38.5	61.5	70.0	95.5	0.005274
13	13.0	37.0	58.5	66.5	92.5	0.038877
14	13.0	38.0	60.0	68.0	93.5	0.005808
15	13.5	37.5	58.5	66.5	92.5	0.042652
16	13.5	37.5	59.0	67.0	92.5	0.011347
17	13.5	38.0	59.5	67.5	93.0	0.002602
18	14.0	38.0	59.0	67.0	92.5	0.013688
19	14.0	38.5	60.0	68.0	93.0	0.029960
20	14.0	39.0	60.5	68.5	93.5	0.034091
21	14.5	38.5	59.5	67.5	92.5	0.019926
22	14.5	39.0	60.0	68.0	93.0	0.031384

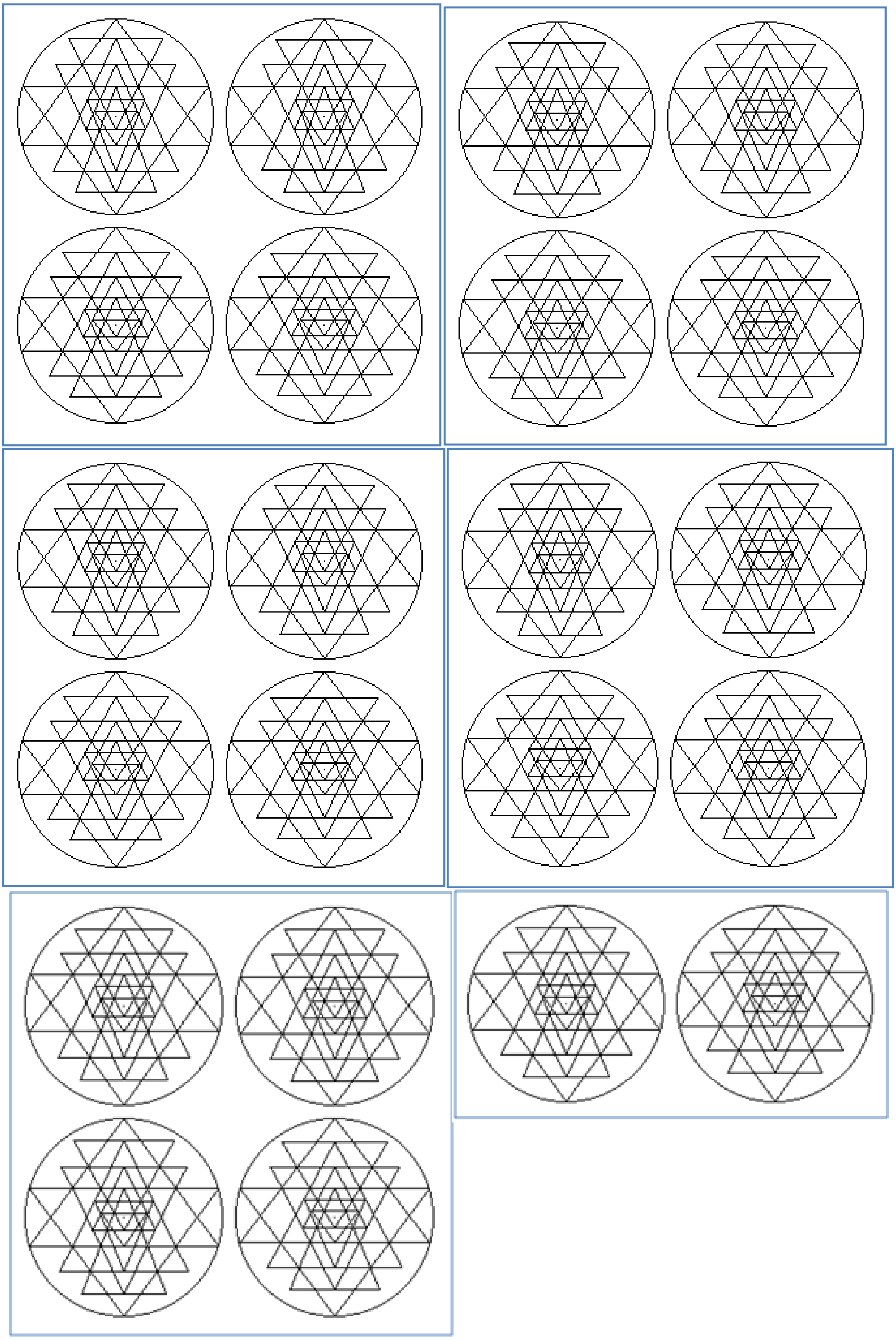


Fig.5.2. Twenty-two valid and aesthetically appealing drawings of Srichakra based on data given in Table 5.1

5. 3. Sensitivity to Input Specifications

In Sec.3.6 we referred to the sensitivity of closing error to variation in the input data. In order to study this aspect of sensitivity a little deeper we did the following experiment. Let us select one of the solutions. For example, Case 2 in the above Table 5.1. The range of search for valid solutions is now set to the values around this solution as follows:

AZ = 11.4 to 11.6 AF = 36.9 to 37.1
AY = 59.9 to 60.1 AC = 68.4 to 68.6
AX = 94.9 to 95.1

Also, the resolution (step size) is set to be 0.05 unit. The input data for the worst and best cases of closing error are compared with the data of Case.2 (shown in bold) in the following Table.

AZ	AF	AY	AC	AX	Closing Error %
11.60	36.90	60.10	68.40	95.05	0.236
11.50	**37.00**	**60.00**	**68.50**	**95.00**	**0.029634**
11.45	37.10	60.00	68.45	94.95	0.0000092

The worst case gives a closing error of 0.236% (noticeable closing error) where four of the input data points differ by 0.1 and one of the data points differs by 0.05. The best case gives a closing error of 0.0000092% (practically zero) where one of the data points differ by 0.1, three of the data points differ by 0.05 and there is no change in one data point. Thus, even a small change in the input data causes a large change in the closing error.

The experiment is repeated now with a resolution of 0.005 unit with reference to the above best case. The results are shown in the following Table.

AZ	AF	AY	AC	AX	Closing Error %
11.440	37.105	59.990	68.460	94.940	0.02
11.450	**37.100**	**60.000**	**68.450**	**94.950**	**0.0000092**
11.445	37.100	60.005	68.455	94.955	0.0000089

The worst case error increases by a factor of about two thousand for changes as small as 0.01 in four of the input data points and a change of about .005 in one input data point. There is no significant change in the best case.

For a very accurate drawing of *Srichakra* markings have to be done to a very high resolution. One has to make use of computers and sophisticated plotters to generate and draw *Srichakra* to a very high accuracy, if so desired.

5. 4. Invariant Relation in *Srichakra*

We mentioned that there are 826 theoretically valid solutions when searched for a range of input data as in Sec 4.4.1, for a resolution of 0.5 unit with a closing error less than 0.05% and no base errors and no Ashtakona error. In fact there are a large number of solutions for *Srichakra* for other ranges and resolutions. In order to refer to these solutions by the generic name of *Srichakara,* there has to be an underlying invariant pattern in all these valid drawings. What is the underlying invariant pattern in *Srichakra* solutions apart from the fact that there are five upright and four inverted overlapping triangles?

5.4.1. Relative Area Ratios

During our investigations we noticed that in the *Chatur-dashara*, whenever the area of the triangle Z1NZ2 increased the area of triangle N1GN decreased (See Fig.2.1). Similarly there is a trade-off of the areas between triangles X1LX2 and L1HL (See Fig.2.1). This gave us a hint to study the total area of all the triangles within a *chakra (Fig.1.4)*. In *Bhu-prastara* representation of *Srichakra*, the triangles in each *chakra* is shown filled (See cover page drawing) and in a *meru Srichakra*, these *chakras* are represented at different heights. One may anticipate that the total area of all the triangles in each *chakra* may have some significance.

Let A14 be the sum of the areas of fourteen triangles of *Chatur-dashara* (Fig.5.3). Let B10 be the sum of the areas of ten triangles of *Bahya-dashara.* Let A10 be the sum of the areas of ten triangles of *Antar-dashara.* Let A8 be the sum of the areas of eight triangles of *Ashtakona*. Let A3 be the area of the innermost inverted triangle (*Yoni*).

We have added two more triangles to the *chatur-dashara chakra* to yield a total of 18 triangles, which we refer to as *Ashta-dashara* and denote by A18. *Ashta-dashara* is not a *chakra* and this term is not found in the conventional literature. It is a term we have introduced here. The triangles numbered 15-18 (shown in Blue) as well as six other triangles (shown in Red in Fig.5.3) are not included in the 43-triangles complex traditionally associated with Srichakra since these triangles and the rhombic shaped areas (in White) correspond to the gaps or space between two successive levels in a *Meru Srichakra.*

The sum of the areas are computed for the 22 solutions (Table 5.1) and the results are shown in Table 5.2. The mean and the standard deviation are also shown in the Table. The ratio of the standard deviation to the mean is very small and it varies between 0.32% to 1% (i.e., 0.0032 to 0.01). This implies that there is only a slight variation in the total areas though the *Srichakra* solutions are obtained for different sets of input data (Table.5.1)..

Square root of the total areas gives us a better insight. Square root of an area may be interpreted as the side of an equivalent square with the same area. Let SA18, SA14, SB10, SA10, SA8 and SA3 be the square root of the areas A18, A14, B10, A10, A8 and A3, respectively. The results are shown in Table 5.3. These results by themselves don't explicitly give us any interesting insight.

In the next step, we compute the relative total areas rather than the actual total areas as the latter depends on the size (AD=108) of *Srichakra*. The relative areas are obtained by normalizing the areas with respect to the square-root of the *Bahya-dashara chakra*, SB10. *Bahya-dashara* has been chosen as the reference for normalization as it is the fourth *chakra* (central *chakra* of the seven *chakras*). The normalized areas are NA18=SA18/SB10, NA14=SA14/SB10, NA10 = SA10/SB10, NA8 = SA8/SB10 and NA3 = SA3/SB10. Since the areas A18 and A14 are greater than B10, NA18 and NA14 are expected to be greater than unity.

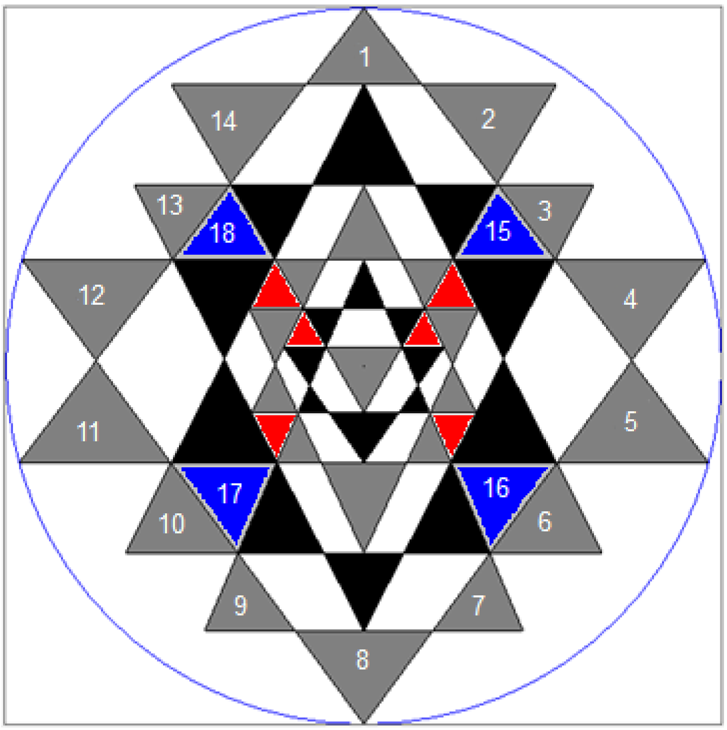


Fig.5.3. Total areas of various chakras. Four (15-18) additional triangles (in Blue) have been added to yield Ashta-dashara group.

We add one more area. Let AO be the area of the inner-most circle. Since R0=AD/2 (=54), the area of circle AO (9160.88) is the same for all the 22 solutions. Square root of AO is denoted by SAO (95.712). In order to study *indirectly* the variation in SB10 we normalize SAO by SB10 to obtain NAO=SAO/SB10. NAO is expected to be greater than unity, since the area of the circle, AO, is greater than B10. Since AO is a constant across 22 solutions, changes in NAO imply changes in B10.

Table 5.4 shows the normalized areas. The standard deviation is extremely small. It is amazing to note that the proportion of the areas remains almost the same irrespective of the wide changes in the input data. The mean, the standard deviation and the ratio of the standard deviation relative to the mean are computed for all the 826 solutions (See Table 5.5). The results are similar to those obtained for the 22 solutions. In other words there appears to be an underlying invariant relation amongst the total areas for all the valid solutions of *Srichakra*.

Discussion of the Results: The normalized area NAO is close to 3 (with a standard deviation of 0.003). It implies that the sum of the areas of triangles in *bahya-dashara* (B10) is about one-ninth the area of the inner-most circle. NA18 is close to 1.5 (3/2), which implies that the area of *Ashta-dashara* is one-fourth the area of the inner-most circle. NA14 is about 1.3.

Mean NAO : Mean NA18 : Mean NA14 :: 3 : 1.5 : 1.3

An interesting result is noted with respect to NA10, NA8 and NA3, which is discussed in the next sub-section.

5. 4. 2. Relation of Some Area Ratios to Golden Ratio

The so called Golden Ratio is represented by the Greek symbol ϕ (PHI, pronounced as 'fee'). Like π (PI), PHI is an irrational number

PHI = 1.61803398... = 1.618 (to the first three decimal places)

The reciprocal of PHI, Phi, is

Phi = 1/PHI = (PHI-1) = 0.61803398.... = **0.618** (to first three decimal places)
Phi-squared = Phi*Phi = 1 - Phi = 1-0.618 = **0.382**
Phi-cubed = 0.382 * 0.618 = Phi - Phi-Square = 0.618-0.382 = **0.236**

Geometrically, PHI is obtained as one-half unit plus one-half of the hypotenuse of a right angled triangle with the lengths of the sides being 1 and 2 units (i.e., 1/2 + 1/2 of square root of 5). Another way to obtain PHI is using Fibonacci sequence, a sequence of numbers with a special property. Each number in the Fibonacci sequence is the sum of the two preceding numbers, initialized with 1-1 (1-1-2-3-5-8-13-21-34...). The ratio of successive numbers (21/13, 34/21,...) in Fibonacci sequence converges to PHI. The Golden Ratio is also called Divine Proportion. The number PHI manifests abundantly in Nature. Information on PHI is easily available on the Web for an interested reader.

It is interesting to note the following relation (See Table 5.5):

Mean NA10 : Mean NA8 : Mean NA3 :: 0.619 : 0.409 : 0.230

which is very close to

Phi : Phi-squared : Phi-cubed :: **0.618 : 0.382 : 0.236**.

5. 4. 3. Another Hypothesis on *Srichakra* and the Golden Ratio

There exists yet another hypothesis relating *Sriyantra (Srichakra)* and the Golden Ratio. For an English version: visit URL: http://eye-of-revelation.org/PDF/THEORY-SriYantra.pdf, a work that seems to originate in 2003. In that work, the hypothesis is that the three concentric circles with diameters equal to (AD)(Phi), (AD)(Phi-Squared) and (AD)(Phi-Cubed) are closely related to *Srichakra*. We have tested this hypothesis with a *Srichakra* constructed using the traditional data (Sec 3.1), since that data gives a nearly symmetric *Srichakra* drawing. *Srichakra* along with the hypothesized three concentric circles is shown in Fig.5.3. The lines AB (and AB') as well as the line DE (and DE') are supposed to be tangential to the first concentric circle (shown in Red). However, we notice that there is a slight gap between the inclined lines AB (AB') and the first concentric circle. The gap is larger between DE (DE') and the expected tangent to the first concentric circle. The second concentric circle doesn't show any interesting relation except that it seems to pass through the point I (I'). The inclined line MP (MP') is nearly, but not exactly, tangential to the third concentric circle. This author is of the opinion that the normalized area ratios show a better correspondence to Phi and its powers rather than these concentric circles.

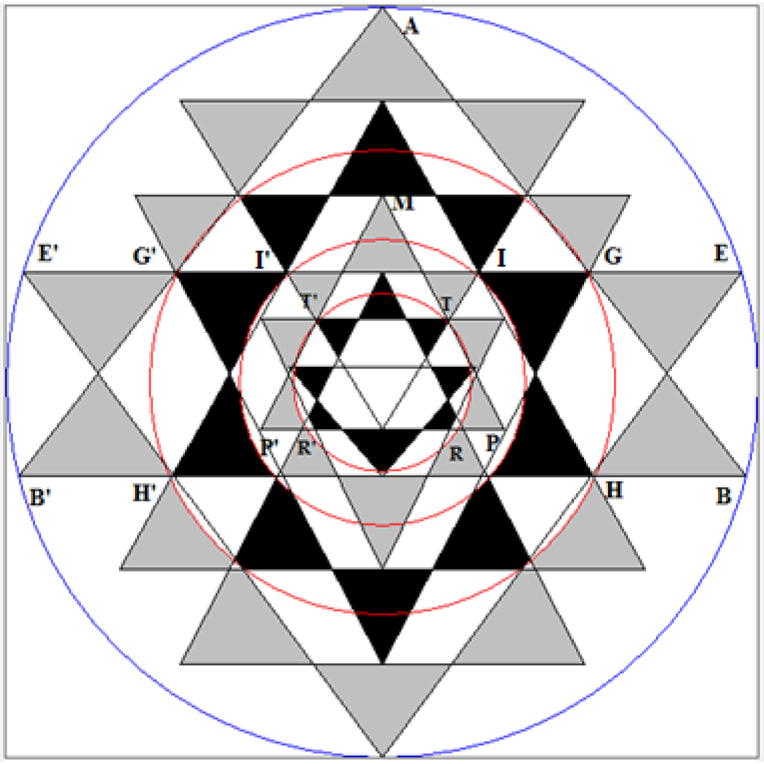


Fig. 5.3. Relationship between Srichakra of Radius 'R' and three concentric circles (shown in Red) of radii (R)(Phi), (R)(Phi-squared) and (R)(Phi-cubed)

Table 5.2

Sum of the Areas of Triangles belonging to

Ashta-dashara (A18), Chatur-dashara (A14), Bahya-dashara (B10), Antar-dashara (A10), Ashtakona (A8) and Yoni (A3)

Sl No	A18	A14	B10	A10	A8	A3
1	2198.66	1799.91	973.49	371.83	160.51	53.07
2	2190.08	1793.38	976.98	374.14	163.09	52.64
3	2192.62	1792.84	974.63	372.82	161.96	52.67
4	2185.34	1788.16	977.62	374.91	164.63	52.16
5	2187.47	1786.90	975.56	373.75	163.49	52.24
6	2190.43	1794.60	977.47	374.13	164.19	52.36
7	2184.00	1784.46	978.85	376.21	160.40	53.00
8	2181.59	1784.16	978.05	375.65	166.25	51.66
9	2181.68	1775.71	974.40	374.08	164.34	51.64
10	2183.25	1782.12	976.28	374.64	165.08	51.77
11	2185.85	1789.52	978.25	375.09	165.79	51.90
12	2187.73	1788.09	975.74	373.60	164.51	51.92
13	2181.55	1782.29	978.66	376.43	161.78	52.47
14	2184.98	1786.16	979.84	376.51	161.43	52.73
15	2182.26	1788.44	980.83	377.33	163.97	52.05
16	2181.24	1777.74	977.64	376.34	162.19	52.08
17	2182.67	1784.11	979.77	376.91	162.87	52.21
18	2181.39	1783.36	979.47	377.26	164.39	51.66
19	2182.27	1779.52	978.45	376.67	163.21	51.78
20	2183.77	1786.02	980.31	377.03	163.82	51.89
21	2181.44	1779.15	978.20	377.14	164.79	51.24
22	2182.61	1785.38	980.12	377.55	165.40	51.34
Mean	2185.13	1786.00	977.75	375.45	163.55	52.11
S. D.	4.47	5.80	2.03	1.64	1.63	0.50
(S.D./Mean)%	0.20	0.32	0.21	0.44	1.00	0.96

Table 5.3

Square-root of the Sum of the Areas of Triangles belonging to *Ashta-dashara* (*SA18*), *Chatur-dashara (SA14), Bahya-dashara (SB10), Antar-dashara (SA10), Ashtakona (SA8) & Yoni (SA3)*

Sl No	SA18	SA14	SB10	SA10	SA8	SA3
1	46.89	42.43	31.20	19.28	12.67	7.29
2	46.80	42.35	31.26	19.34	12.77	7.26
3	46.83	42.34	31.22	19.31	12.73	7.26
4	46.75	42.29	31.27	19.36	12.83	7.22
5	46.77	42.27	31.23	19.33	12.79	7.23
6	46.80	42.36	31.27	19.34	12.81	7.24
7	46.73	42.24	31.29	19.40	12.67	7.28
8	46.71	42.24	31.27	19.38	12.89	7.19
9	46.71	42.14	31.22	19.34	12.82	7.19
10	46.73	42.22	31.25	19.36	12.85	7.20
11	46.75	42.30	31.28	19.37	12.88	7.20
12	46.77	42.29	31.24	19.33	12.83	7.21
13	46.71	42.22	31.28	19.40	12.72	7.24
14	46.74	42.26	31.30	19.40	12.71	7.26
15	46.72	42.29	31.32	19.43	12.81	7.22
16	46.70	42.16	31.27	19.40	12.74	7.22
17	46.72	42.24	31.30	19.41	12.76	7.23
18	46.71	42.23	31.30	19.42	12.82	7.19
19	46.72	42.18	31.28	19.41	12.78	7.20
20	46.73	42.26	31.31	19.42	12.80	7.20
21	46.71	42.18	31.28	19.42	12.84	7.16
22	46.72	42.25	31.31	19.43	12.86	7.17
Mean	46.75	42.26	31.27	19.38	12.79	7.22
S. D.	0.05	0.07	0.03	0.04	0.06	0.03
(S.D./Mean)%	0.10	0.16	0.10	0.22	0.50	0.48

Table 5.4

Results shown in Table 5.3 are Normalized with respect to SB10.

NAO, normalized area of the inner-most circle is also shown.

Sl No	NAO	NA18	NA14	NA10	NA8	NA3
1	3.068	1.503	1.360	0.618	0.406	0.233
2	3.062	1.497	1.355	0.619	0.409	0.232
3	3.066	1.500	1.356	0.618	0.408	0.232
4	3.061	1.495	1.352	0.619	0.410	0.231
5	3.064	1.497	1.353	0.619	0.409	0.231
6	3.061	1.497	1.355	0.619	0.410	0.231
7	3.059	1.494	1.350	0.620	0.405	0.233
8	3.060	1.493	1.351	0.620	0.412	0.230
9	3.066	1.496	1.350	0.620	0.411	0.230
10	3.063	1.495	1.351	0.619	0.411	0.230
11	3.060	1.495	1.353	0.619	0.412	0.230
12	3.064	1.497	1.354	0.619	0.411	0.231
13	3.060	1.493	1.350	0.620	0.407	0.232
14	3.058	1.493	1.350	0.620	0.406	0.232
15	3.056	1.492	1.350	0.620	0.409	0.230
16	3.061	1.494	1.348	0.620	0.407	0.231
17	3.058	1.493	1.349	0.620	0.408	0.231
18	3.058	1.492	1.349	0.621	0.410	0.230
19	3.060	1.493	1.349	0.620	0.408	0.230
20	3.057	1.493	1.350	0.620	0.409	0.230
21	3.060	1.493	1.349	0.621	0.410	0.229
22	3.057	1.492	1.350	0.621	0.411	0.229
Mean	**3.061**	**1.495**	**1.352**	**0.620**	**0.409**	**0.231**
S. D.	0.003	0.003	0.003	0.001	0.002	0.001
(S.D./Mean)%	0.104	0.186	0.217	0.137	0.493	0.494

Table 5.5

Results for 826 Theoretically Valid Solutions

	NAO	NA18	NA14	NA10	NA8	NA3
Mean	**3.047**	**1.495**	**1.356**	**0.619**	**0.409**	**0.230**
S. D.	**0.019**	**0.006**	**0.008**	**0.002**	**0.016**	**0.013**
(S. D./Mean)%	**0.65**	**0.39**	**0.58**	**0.32**	**3.9**	**5.6**

6

Two Circles Hypothesis

6.1 Motivation and Background for Proposing the Hypothesis

In Chapter 3, it is noted that there is a 'gross error' (in Q1) and a 'noticeable closing error' (1.248%) in the *Srichkara* drawn using the traditional data (Table 3.1) given by Sri Kaivalyashrama or Sri Lakshmidhara. However, the *Ashtakona* of *Srichakra*, based on the traditional data has an aesthetical appeal. Also, the width ZZ1 is the same as the width XX1. Similarly, the width MN1 is nearly same as the width KL1. These further enhance the aesthetic appeal. The author wondered if there exists an alternate method for constructing *Srichakra* that retains the aesthetic appeal seen in the *Srichakra* constructed using the traditional data while at the same time reducing the closing error.

The proposed two circles hypothesis arose In the course of author's research relating to the geometry of '*Sri Sudarshana Mudra'*, a mystical drawing, drawn in 1947 by Sri Sriranga Saduguru (1917-1969). Visit ayvm.in for more details on Sriranga Sadguru. It is the author's conjecture that in the mystical drawing of *Sudarshana Mudra* what *appears* to be a single circle is in fact the joining of upper-half and lower-half of two different circles with the same radii but displaced centres. Inspired by this observation, the author is proposing two circles hypothesis for constructing *Srichakra*.

6. 2. Construction of *Srichakra* based on Two Circles Hypothesis

The same nomenclature as in Chapter 2 is followed here. Some additional nomenclature is added. We follow the method proposed by MPS, i.e., the use of five input data points specification. Draw a circle of diameter of 48 units as in the traditional data (centre at x=0, y=24 units). This is shown by solid line in Fig.6.1a. We refer to this as the **reference circle**. Shift the centre of the circle upwards by an offset=2.4 units (x=0, y=24-2.4 = 21.6 units). This offset of 2.4 units has been deduced by searching for the lowest closing error over a wide range of values for the displacement of the centre. Draw another circle of the **same diameter** (48 units) but with the shifted centre, referred to as the **shifted circle**. This is shown by dashed line in Fig.6.1a. In the final drawing, only the upper-half of the reference circle and the lower-half of the shifted circle are retained as shown in Fig.6.1b; To a casual observer, this drawing (Fig.6.1b) appears to be a single circle.

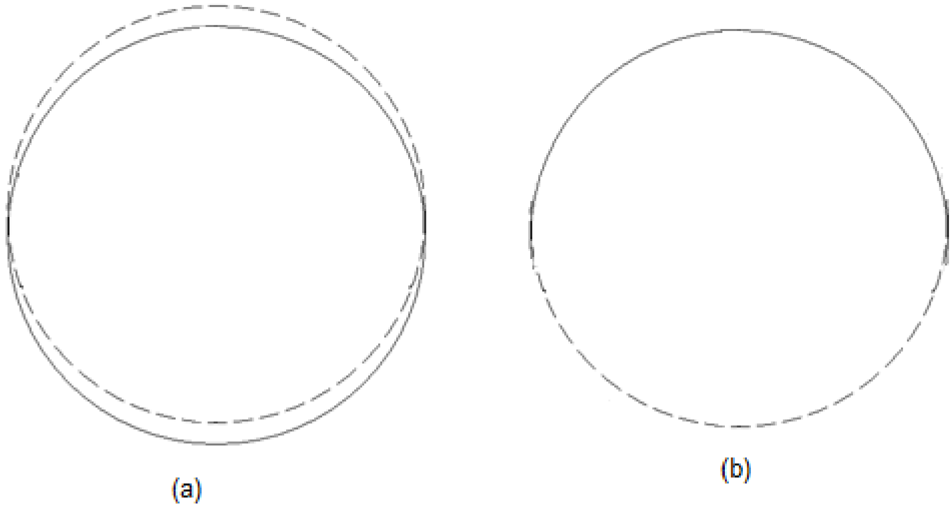


Fig.6.1. (a) The reference circle in solid line and the shifted circle in dashed line. The shifted circle has its centre moved upwards (by 2.4 units) relative to the reference circle but has the same diameter (48 units) as that of the reference circle.

(b) Only the upper-half of the reference circle and the lower-half of the shifted circle are retained in the final drawing. To a casual observer, this drawing appears as if it is a single circle.

The vertex 'A' lies on the **reference circle**. Mark the input data of points on the vertical axis as AZ (=6 units), AF (=17 units), AY (=27 units), AC (=30 units) and AX (=36 units), which remain the same as in the traditional data. The horizontal chord at F meets the circle at E, which lies on the reference circle.

The lowest point lies on the shifted circle and is denoted by D2 (See Fig.6.2b). The location of the right extreme point of the chord through C now changes to B2, which now lies on the shifted circle. The chord CB2 belongs to the shifted circle. In order to find the coordinates of B2, let A2 be the vertex (top most point) on the shifted circle. A2 is not seen in the final drawing. Note that the product (CB2)(CB2) = (A2C)(CD2). Once the point B2 is determined, rest of the procedure to construct *Srichakra* remains similar to that described in Ch. 2.

The drawing of *Srichakra* constructed with the two circles hypothesis is shown in Figs. 6.2a and 6.2b. Without a careful measurement it is difficult to ascertain that what appears to be a single circle is in fact made up of two circles. This can be compared with the *Srichakra* shown in Fig.3.1a. The areas of the filled triangles of *Ashtakona* are well proportioned as in the case of Fig.3.1a. The drawing appears to be aesthetically appealing. No gross error or closing error is noticeable even in an enlarged view (Fig.6.2c). When the traditional data is used along with this two-circles hypothesis, we not only get an aesthetically appealing *Srichakra*, as obtained by the traditional method, but also a very low closing error (0.002453%).

Further, *Srichakra* constructed based on two circle hypothesis also satisfies the invariant relation of the area ratios (Sec. 5.6):

Mean NAO : Mean NA18 : Mean NA14 :: 3.01 : 1.50 : 1.35

Mean NA10 : Mean NA8 : Mean NA3 :: 0.619 : 0.391 : 0.236

Here, the area of the circle, A0 is computed as one-half of the area of circle with radius AD/2 plus one-half of the area of circle with radius (AD/2 - Offset).

Given the above result. one wonders if traditional *Srichakra* drawing could have been constructed based on two circles hypothesis. Also, this result shows that alternate solutions for constructing *Srichakra* may exist.

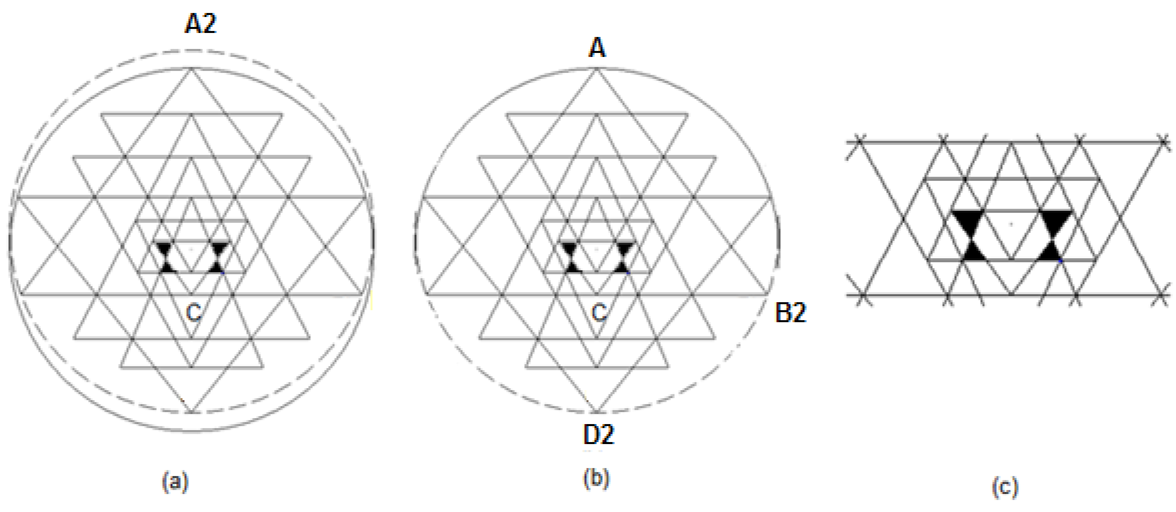


Fig.6.2. (a) Srichakra constructed based on the proposed two circles hypothesis and five input points of traditional data. (b) Top-half of the reference circle and bottom-half of the shifted circle are retained in the final drawing. (c) An enlarged view doesn't show any noticeable gross or closing error. Filled triangles show a balanced proportion of the areas of some of the Ashtakona triangles as in traditional drawing.

7

Conclusion

We have presented a clearly defined and well illustrated procedure for constructing *Srichakra*. We acknowledge the fact that there is a need to specify only five input data points. We have shown that multiple solutions are possible. We summarize the following main results.

(a) Theoretical solution for *Srichakra* with zero closing error may exist but it may require infinite precision in specifying the input data. It may not be possible to physically construct such a *Srichakra* due to practical difficulties.

(b) Implementation of near ideal solution requires a very high resolution of the order of 0.001 units or better, which is practically impossible

(c) In a drawing of *Srichakra*, when the closing error is less than the line thickness (or one pixel resolution) then the closing error can be deemed to be practically zero.

(d) Practically speaking, multiple solutions are possible. There are as many as 826 valid solutions for a resolution of 0.5 unit with a closing error less than 0.05% and with no base error. Of these 826 solutions, 22 are also aesthetically appealing.

(e) It has been noted that the total area of the triangles of the five *chakras:, Chatur-dashara*, *Bahya-dashara*, *Antar-dashara, Ashtakona* and *Yoni* bear constant ratios (near invariance) for the valid solutions of *Srichakra*. The total areas of *Antar-dashara, Ashtakona* and *Yoni* relative to the total area of *Bahya-dashara* are close to Phi, its square and its cube, respectively. An open research problem is to derive the input data given the invariant relation.

(f) *Srichakra* construction is highly sensitive to input data and even a small error in the marking may result in noticeable gross errors.

(g) Two circles hypothesis also provides us with an aesthetically appealing drawing with a very low closing error. Alternate solutions thus seem possible.

Many of the open issues may be resolved if photographs or copies of traditionally worshipped *Srichakra* used in temples are made available for research.

Bibliography

Alexi Pavlovich Kulaichev, "*Sriyantra* and its mathematical properties", in *Indian Journal of History of Science*, vol.19 (3), pp. 279-292, 1984.

Alexi Pavlovich Kulaichev and Dina Mikhalovna Ramendic, "*Sriyantra* - The ancient instrument to control the psychophysiological state of man", in *Indian Journal of History of Science*, vol.24 (3), pp. 137-149, 1989.

Bhaskararaya, *Varivasysa Rahasya and Its Commentary*, Adyar Library and Research Centre. First Edn. 1934. Fourth Edn. 1976. Edited with English Translation by S. Subrahmanya Sastri.

Gopinath Rao, T. A., *Elements of Hindu Iconography*. Law Printing Press, Madras, 1914.

Jakkanamatya's Commentary on Saundaraya Lahari in Kannada, Ed. S. Narayanaswamy Sastry, Oriental Research Institute, Mysore University, First Edn. 1959.

Ramachandra Rao S. K., *Sri-Chakra : Its Yantra, Mantra and Tantra*, Kalpatharu Research Academy, Bangalore, 1982. First Edn.

Satyanarayana, R., "*Sri Vidya Navavarana"*, (in Kannada) Article appeared in *Sri Matha*, Sri Kamakshi Sharadamba DevAshtana Kodanda Ashrama Mutt, Hebbur, Tumkur, 1996. pp. 21-33.

Saundarya Lahari of Sri Sankaracarya with the commentary of Lakshmidhara - Oriental Research Institute, University of Mysore, (in Sanskrit), 4th Revised Edn., 1969. Ed. Venkata-nathacharya, N.S.

Saundarya Lahari including commentary by Sri Sadashivnandanatha as in Samyamodhini (in Telugu) Ed. T. Ramalingeswar Rao Sharmagaru, Tagore Publishing House, Hyderabad, 1995.

Shankarnarayanan, M. P., *"An accurate method of construction of Sri Chakra"* - (Publication date, Journal and Author's address have not been given in the paper.)

Shankaranarayananan, S., *Sri Chakra*, Samta Books and Sri Mira Trust, Chennai. First Edn. 1971. Samata Edn. 1996.

Sreenivasarao's blogs (srinivasaraos.com/tag/sri-yantra).

Subrahmanya Sastri S., and Srinivasa Ayyangar T.R., *The Ocean of Beauty - Saundarya Lahari*, The Theosophical Publishing House, Chennai, 8th Reprint, 1997. First Edn. 1937.

Suryanarayana Murthy, C., *Sri Lalita Sahasranama*, Bharitiya Vidya Bhavan, Bombay. Sec Revised Edn. 1975.

Umanandanatha, *Nityotsava*, Gaekwad's Oriental Series, No. XXIII, Baroda Oriental Institute, 1930. Ed. A. Mahadeva Sastri. Re-edited by: Swami Trivikrama Tirtha.

URL: http://eye-of-revelation.org/PDF/THEORY-SriYantra.pdf

Woodroffe, Sir John, *The Garland of Letters*, Ganesh & Co, Madras, 1963. pp.240-242.

APPENDIX-A
COMPUTER PROGRAM LISTING
for
CALCULATING and PLOTTING *SRICHAKRA*

' Program Listing in QuickBasic Ver 4.0 (DOS Version)

```
        'Specify the Input Data Here:
        AZ = 6: AF = 17: AY = 27: AC = 30: AX = 42: AD = 48

        '-------------------------------------------OTHER DATA ------------------------------
' Data used for MPS's method Fig.3.2
'AZ = 6: AF = 17: AY = 27: AC = 30: AX = 42: AD = 48

'MPS's Data used for Fig.3.4
'AZ = 12.05: AF = 36.2: AY = 55.48: AC = 62.87: AX = 87.7: AD = 100!

'TVA and MNR Data approximation to MPS
'AZ = 13: AF = 39: AY = 60: AC = 68: AX = 94: AD = 108     'Fig.3.5
'AZ = 13: AF = 39: AY = 60: AC = 68: AX = 94.5: AD = 108   'Fig.3.6
'AZ = 13: AF = 39: AY = 60: AC = 68: AX = 95: AD = 108

'Resolution = 0.5, Very Low Error. Data, Z1 is interior used for Fig. 3.7
AZ = 13.5: AF = 38: AY = 60: AC = 68.5: AX = 94: AD = 108

'Judgement based on Low Closing Error could be misleading.
' Data used for Fig.3.8
'AZ = 13.5: AF = 40: AY = 59.5: AC = 68: AX = 96: AD = 108

'Error in ZZ1 though Closing Error = 0.0015%
'AZ = 11: AF = 39: AY = 59: AC = 67: AX = 96: AD = 108

'Author's preferred choice      Closing error = 0.009336%
'AZ = 11.5: AF = 38: AY = 61.00: ‘AC = 68.5: AX = 94: AD = 108
'------------------------------------------------------------------------
'Some Examples of very closing error with a resolution of 0.01 units
'Data used for Fig.5.1a :       Closing error = 0.0000076%
'AZ = 13.35: AF = 39.40: AY = 59.55: AC = 67.00: AX = 93.20: AD = 108

'Data used for Fig.5.1b :       Closing error = 0.000025%
'AZ = 12.45: AF = 38.95: AY = 60.25: AC = 67.55: AX = 93.35: AD = 108

'Data used for Fig.5.1c :       Closing error = 0.000046%
'AZ = 13.45: AF = 38.00: AY = 59.95: ‘AC = 68.10: AX = 93.40: AD = 108

'Data used for Fig.5.1d :       Closing error = 0.00005%
'AZ = 13.80: AF = 38.10: AY = 60.00: ‘AC = 68.10: AX = 93.10: AD = 108

'--------------------- End of Data --------------------------------------------------
```

```
'---------------Program to calculate and draw Sirchakra begins here----------------
            CLS
            'VGA Screen
            SCREEN 12

            'Coordinates of the Input Data
            yA = (AD / 2): xA = 0
            yD = (AD / 2 - AD): xD = 0:

            yZ = (AD/2 - AZ): xZ = 0: yF = (AD/2 - AF): xF = 0
            yY = (AD / 2 - AY): xY = 0: yC = (AD / 2 - AC): xC = 0
            yX = (AD / 2 - AX): xX = 0

            yE = yF: xE = SQR(AF * (AD - AF))
            yB = yC: xB = SQR(AC * (AD - AC))

            'find xG,yG     Xn of AB and  FE
            mAB = (yB - yA) / (xB - xA): cAB = yA
            mFE = 0: cFE = yF
            xG = -(cAB - cFE) / (mAB - mFE)
            yG = mAB * xG + cAB

            'find xH and yH from Xn of DE and CB
            mCB = 0: cCB = yC
            mDE = (yE - yD) / (xE - xD): cDE = yD
            xH = -(cDE - cCB) / (mDE - mCB)
            yH = mDE * xH + cDE

            'find xI,yI from Xn of ZH and FE
            mZH = (yZ - yH) / (xZ - xH): cZH = yZ
            mFE = 0: cFE = yF
            xI = -(cZH - cFE) / (mZH - mFE)
            yI = mZH * xI + cZH

            'Find ZZ1: From similar triangles YIF and YZ1Z
            xZ1 = (yZ - yY) * (xI / (yF - yY))
            yZ1 = yZ

            'find xN,yN  from  Xn of AB and YZ1
            mAB = (yB - yA) / (xB - xA): cAB = yA
            mYZ1 = (yZ1 - yY) / (xZ1 - xY): cY = yY
            xN = -(cAB - cY) / (mAB - mYZ1)
            yN = mAB * xN + cAB

            'Find xJ, yJ : from Intersection of  XG and CB
            mCB = 0: cCB = yC
            mXG = (yX - yG) / (xX - xG): cXG = yX
            xJ = -(cCB - cXG) / (mCB - mXG)
            yJ = mCB * xJ + cCB
```

```
'find xM, yM  :
'Horizontal through N: slope=0:
xM = 0: yM = yN

'extend MN upto N1:
'Find xN1,yN1 based on similar triangles XMN1 & XFG
xN1 = (yM - yX) * (xG / (yF - yX))
yN1 = yM

'Find XX1
' From similar triangles XFX1 and CFJ
xX1 = (yF - yX) * (xJ / (yF - yC))
yX1 = yX

'find L : Intersection of FX1 and and DE
mFX1 = (yX1 - yF) / (xX1 - xF): cFX1 = yF
mDE = (yE - yD) / (xE - xD): cDE = yD
xL = -(cFX1 - cDE) / (mFX1 - mDE)
yL = mFX1 * xL + cFX1

'find xQ and yQ from Xn of YN  and FJ
mYN = (yN - (yY)) / (xN - xY): cYN = yY
mFJ = (yJ - yF) / (xJ - xF): cFJ = yF
xQ = -(cYN - cFJ) / (mYN - mFJ)
yQ = mYN * xQ + cYN

'Horizontal from L meets AD at K
yK = yL: xK = 0

'extend FL to FL1 till Hzl through KL reaches L1
'use similar triangles ZCH, ZKL1
xL1 = (yZ - yK) * (xH / (yZ - yC))
yL1 = yK

'find R as the point of intersection of Hzl through Y and FX1
        ‘YR/FY = xX1/FX
       xR = (yF - yY) * (xX1 / (yF - yX))
yR = yY

'Hzl through Y to meet XG meets at P ; gives MP
'similar triangles XYP and XFG   (YP/YX)=(FG/FX)
xP = (yY - yX) * (xG / (yF - yX))
yP = yR

'find xT1 and yT1 from Xn of YZ1  and MP
mYZ1 = (yZ1 - yY) / (xZ1 - xY): cYZ1 = yY
mMP = (yP - yM) / (xP - xM): cMP = yM
xT1 = -(cYZ1 - cMP) / (mYZ1 - mMP)
yT1 = mYZ1 * xT1 + cYZ1
```

```
'Hzl through Q meets Vertical Axis at U
xU = 0: yU = yQ

'join UQ and produce to meet MP at Q1
'Hzl through Q meets MP at Q1
'use similar triangles YPM and UQ1M
'(UQ1/MU)=(YP/MY)
xQ1 = (yM - yU) * (xP / (yM - yY))
yQ1 = yQ

'Hzl through T1 meets Vertical Axis at S
xS = 0: yS = yT1

'produce ST1 to meet ZH at T
'similar triangles ZST and ZCH
'(ST/ZS)=(CH/ZC)
xT = (yZ - yS) * (xH / (yZ - yC))
yT = yT1

'find xQ2 and yQ2 from Xn of TK  and  MP
mTK = (yK - yT) / (xK - xT): cTK = yK
mMP = (yP - yM) / (xP - xM): cMP = yM
xQ2 = -(cTK - cMP) / (mTK - mMP)
yQ2 = mTK * xQ2 + cTK

R0 = AD / 2
VIEW (20, 20)-(280, 280)
WINDOW (-R0, -R0)-(R0, R0)
GOSUB plot

‘For seeing  part of Srichakra in an enlarged scale
        VIEW (20, 300)-(280, 460)
WINDOW (0, yY - 10)-(xT, yT)
GOSUB plot

LOCATE 2, 40: PRINT "Errors if any:"
' Error Checking
'Is Zarc < xZ1
Zarc = SQR(AZ * (AD - AZ))
IF xZ1 > Zarc THEN
LOCATE 4, 45: PRINT "ERROR in ZZ1"
END IF

'Is Marc < MN1
AM = (yA - yM)
Marc = SQR(AM * (AD - AM))
IF (xN1 > Marc) THEN
      LOCATE 6, 45: PRINT "Error in MN"
END IF
```

```
'Is Karc < xL1 then error
AK = yA - yK
Karc = SQR(AK * (AD - AK))
IF xL1 > Karc THEN
        LOCATE 14, 45: PRINT "Error in KL"
END IF

'is Xarc < xX1
Xarc = SQR(AX * (AD - AX))
IF xX1 > Xarc THEN
        LOCATE 16, 45: PRINT "Error in XX1"
END IF

IF xQ2 < xQ1 THEN
        LOCATE 10, 45: PRINT "Error in Ashtakona"
END IF

'find R1 intersection of TK and FJ
         'find xR1 and yR1 from Xn of TK  and FJ
mTK = (yK - yT) / (xK - xT): cTK = yK
mFJ = (yJ - yF) / (xJ - xF): cFJ = yF
         xR1 = -(cTK - cFJ) / (mTK - mFJ)
yR1 = mTK * xR1 + cTK

'does R1 and R coincide
        clerr = (xR - xR1) ^ 2 + (yR - yR1) ^ 2
IF clerr > 0 THEN
        clerr = SQR(clerr)
ELSE
        clerr = 0
END IF
LOCATE 20, 40: PRINT "Input Data:"
LOCATE 21, 40:
     PRINT USING "AZ = ##.##  AF = ##.##  AY = ###.##"; AZ; AF;AY
LOCATE 22, 40:
     PRINT USING "AC = ##.##  AX = ##.##  AD = ###"; AC; AX; AD;
LOCATE 24, 40:
     PRINT USING "Closing Error = #.######%"; 100! * (clerr / AD);


x$ = INPUT$(1)

END



plot:
        CIRCLE (0, 0), R0
        PSET (xZ, yZ)
```

```
PSET (xF, yF)
PSET (xY, yY)
PSET (xC, yC)
PSET (xX, yX)

LINE (xF, yF)-(xE, yE)              'FE
LINE (xC, yC)-(xB, yB)              'CB
LINE (xA, yA)-(xB, yB)              'AB
LINE (xD, yD)-(xE, yE)              'DE
LINE (xZ, yZ)-(xH, yH)              'ZH
LINE (xY, yY)-(xI, yI)              'YI
LINE (xZ, yZ)-(xZ1, yZ1)            'ZZ1
LINE (xI, yI)-(xZ1, yZ1)            'IZ1
LINE (xX, yX)-(xG, yG)              'XG
LINE (xM, yM)-(xN, yN)              'MN
LINE (xN, yN)-(xN1, yN1)            'NN1
LINE (xG, yG)-(xN1, yN1)            'GN1
LINE (xF, yF)-(xJ, yJ)              'FJ
LINE (xJ, yJ)-(xX1, yX1)            'JX1
LINE (xX, yX)-(xX1, yX1)            'XX1
LINE (0, yK)-(xL, yL)               'KL
LINE (xL, yL)-(xL1, yL)             'LL1
LINE (xH, yH)-(xL1, yL1)            'HL1
LINE (xY, yY)-(xR, yR)              'YR
LINE (xR, yR)-(xP, yP)              'RP
LINE (xP, yP)-(xM, yM)              'PM
LINE (xU, yU)-(xQ, yQ)              'UQ
LINE (xQ, yQ)-(xQ1, yQ1)            'UQ1
LINE (xQ1, yQ1)-(xC, yC)            'CQ1
LINE (xS, yS)-(xT1, yT1)            'ST1
LINE (xT1, yT1)-(xT, yT)            'TT1
LINE (xT, yT)-(xK, yK)              'TK

'------ Mirror Image ---------------
LINE (xF, yF)-(-xE, yE)             'FE
LINE (xC, yC)-(-xB, yB)             'CB
LINE (xA, yA)-(-xB, yB)             'AB
LINE (xD, yD)-(-xE, yE)             'DE
LINE (xZ, yZ)-(-xH, yH)             'ZH
LINE (xY, yY)-(-xI, yI)             'YI
LINE (xZ, yZ)-(-xZ1, yZ1)           'ZZ1
LINE (-xI, yI)-(-xZ1, yZ1)          'IZ1
LINE (xX, yX)-(-xG, yG)             'XG
LINE (xM, yM)-(-xN, yN)             'MN
LINE (-xN, yN)-(-xN1, yN1)          'NN1
LINE (-xG, yG)-(-xN1, yN1)          'GN1
LINE (xF, yF)-(-xJ, yJ)             'FJ
LINE (-xJ, yJ)-(-xX1, yX1)          'JX1
LINE (xX, yX)-(-xX1, yX1)           'XX1
LINE (xK, yK)-(-xL, yL)             'KL
```

```
LINE (-xL, yL)-(-xL1, yL)            'LL1
LINE (-xH, yH)-(-xL1, yL1)           'HL1
LINE (xY, yY)-(-xR, yR)              'YR
LINE (-xR, yR)-(-xP, yP)             'RP
LINE (-xP, yP)-(xM, yM)              'PM
LINE (xU, yU)-(-xQ, yQ)              'UQ
LINE (-xQ, yQ)-(-xQ1, yQ1)           'UQ1
LINE (-xQ1, yQ1)-(-xC, yC)           'CQ1
LINE (-xS, yS)-(-xT1, yT1)           'ST1
LINE (-xT1, yT1)-(-xT, yT)           'TT1
LINE (-xT, yT)-(xK, yK)              'TK
PSET (0, 0)
RETURN
'----------------------   End   of   Listing   ----------------------------------------
```