MODIFIED CYCLE OF FIFTHS and FOURTHS
FOR KARANATIC MUSICAL SCALE AND RELATED TOPICS

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Abstract

Presently, the well known Pythagorean method of the cycle of Fifths and Fourths, is being used for deriving the intervals of a musical scale. The method has the drawbacks of (a) not resulting in the expected octave at the twelfth step and (b) deriving improper intervals after twelve steps. In this paper the author proposes a modified method of cycle of Fifths and Fourths which overcomes the above drawbacks and correctly derives only the required 22 intervals. Certain complementarities amongst the 22 intervals are demonstrated. It is shown that it is adequate to use either the modified cycle of Fourths or the modified cycle of Fifths instead of using both the methods.

Assuming the intervals of the three swaras, Suddha Madhyama, Prati Madhyama and Panchama, it is shown that all the 22 intervals can be derived without resorting to the cycle of Fifths and Fourths.

1. Introduction

An exhaustive and critical review on the concept of '*sruti'* has been given by Dr R Satyanaraya in his translation and critical commentary on '*Sarnga Deva's Sangeeta Ratnakara*' in Kannada [1]. Various methods of deriving the 'srutis' have been critically reviewed by him in that work. Also there are many interesting websites, for example, [4 and 5]. The author wants to point out to the ocean of information already available instead of replicating the information in this paper. For the benefit of general audience in the conference working definitions of some of the technical terms are given in the Appendix-A.

A certain **reference fundamental frequency** called the *Adhara Shadja* is used as the tonic or reference note in a concert of Karnatic music. An octave is *twice the frequency of Adhara Sadja.* The fundamental frequencies of all other musical notes within an octave (primary octave or '*sthayi*') are then defined relative to the *Adhara Shadja*. This **relative frequency** is usually expressed as **a ratio** (usually of integers). For example, *Panchama* is 3/2 times the *Adhara Shadja*. *Suddha Madhaya* is 4/3 of the *Adhara Shadja*. The ratio is also called the interval. **In this work, we consider *sruti* as the interval between the *Adhara Shadja* and a musical note or swara within the primary octave.**

Generally, in Karnatic music, eleven *swaras* (excluding the tara sadja) within one octave are traditionally recognized. In some ragas, the *sruti* of a particular note may be fine tuned by the artist from its pre-defined position so as to make the raga more appealing. For example, the *Antara Gandhara* has a relative frequency of (81/64) of the *Adhara Shadja*. In order to make the raga more appealing, a different note, an interval of (80/64) is sometimes used in place of *Antara Gandhara*. Where does this new *sruti* of (80/64) come from? Is it a valid *sruti*? Is it a traditionally acceptable *sruti*? Bharata's Natya Shastra mentions 22 *srutis* between Aadhara Sadja ('sa') and its octave (*tara Sadja*). Thus there is scope to modify the eleven *srutis* based on aesthetic appeal provided the new *sruti* so chosen is one of the 22 traditionally known *srutis*. It is important to note that both intervals (81/64) and (80/64) do not occur in the same raga. A particular swara, if fine tuned, must be one amongst the 22 srutis. The problem in musicology is to derive the values of these 22 intervals (or *srutis*).

The problem we are addressing can be explained with an analogy to colours. There are three primary colours. We are allowed to mix these colours and bring out new colours. Assume that we have been given 22 unique colours based on aesthetic appeal, ability to discriminate the colours, and other factors such as tradition. The problem is to find out how the primary colours are mixed to produce these 22 colours. Adhara Sadja, Suddha Madhyama (called the Fourth) and Panchama (called the Fifth) may be regarded as primary notes from which the 22 intervals are to be calculated.

At present, the calculation of 22 intervals (or *srutis*) is based on the Pythagorean Cycle of Fifths and Fourths[1]. Readers may see references [1-6] for discussions on the Cycle of Fifths and Fourths. Cycle of Fifths is attributed to Pythagoras and also referred to as 'Series of Perfect Fifths'. But the method has two serious drawbacks:

(a) At the twelfth step one expects the note to correspond to an octave of the base or tonic note. But this doesn't happen.

(b) Also, after the twelfth step, the method keeps on deriving improper intervals that have to be discarded on an ad hoc basis.

To overcome this problem of off-octave result, there are alternative tuning systems in western music such as equal temperament scale or harmonic scale. Equal temperament scale uses integer powers of the twelfth root of 2. Harmonic scale uses ratios of two integers. Please see website [4] for various tuning systems used in western music.

Karnatic musical scale does not use equal temperament scale. The existing method of the Cycle of Fifths and Fourths is used to predict the 22 *srutis* of Karnatic musical scale despite the drawbacks noted above.

The paper is organized as follows. Firstly, the existing approach of the cycle of Fifths and Fourths is presented. Their drawbacks are discussed. The author[2] proposes a modification of the Cycle of Fifths and Fourths that overcomes the drawbacks in the existing method.

Some additional results are also presented. There are certain complementarities amongst the 22 intervals thereby it is adequate to use either the cycle of Fifths or Fourths. Further it is shown that the 22 srutis can be derived given the three basic swaras: Suddha Madhyama, Prati Madhyama and Panchama.

2. Existing Approach of Cycle of Fifths and Fourths

Cycle of Fifths and Fourths can be explained in two ways: (a) Using ratios of frequencies and (b) Using a logarithmic scale or cent scale.

Cycle of Fifths in Ratio Scale: Consider the *Panchama swara* which has a relative frequency of 3/2 relative to *Adhara Shadja*. Now, *Panchama* of this *Panchama*^{3} is (3/2)(3/2) = (9/4) = (2 times 9)/(2 times 4) = (2 times 9/8) = (9/8 relative to *tara sadja*). Since the numerator is an odd number, multiply and divide the ratio by 2. The number 2 indicates that the derived note corresponds to the upper octave or the *tara sthayi*. The factor 9/8 relative to tara sadja is the same as 9/8 relative to *adhhara sadja*. The ratio 9/8 corresponds to one of the 22 *srutis*. 9/8 corresponds to *Chatusruti Rishabha*.

The procedure can be repeated. *Panchama* of *Panchama* of *Panchama* = (3/2)(3/2)(3/2) = (9/4)(3/2) = (27/8) = (2 times 27/16) = (27/16 relative to *tara sadja*). This derived note corresponds to be one of the 22 *srutis*. 27/16 corresponds to *Chatusruti Dhaivata*. Two steps have been illustrated. The procedure can be continued to any number of desired steps. In other words, use the formula $(3/2)^N$ where N is the step number. Each step gives a new note. In Sec.2, the results are given in cents scale.

Cycle of Fourths in Ratio Scale: Consider the *Suddha Madhyama* whose relative frequency is 4/3. Now, the *Suddha Madhayama* of this *Suddha Madhyama* is (4/3)(4/3)=(16/9). The relative frequency of the new note derived will be (16/9) which corresponds to one of the 22 *srutis*. The relative frequency 16/9 corresponds to *Kaishiki Nishada*.

This procedure can be repeated. *Suddha Madhayama* of *Suddha Madhyama* of *Suddha Madhayama* is (4/3)(4/3)(4/3) = (64/27) = (2 times 32/27) (32/27 relative to tara sadja). The new derived note has relative frequency (32/27) and is one of the 22 *sruits*. 32/27 corresponds to *Sadharana Gandhara*.

Two steps have been illustrated. The procedure can be continued to any number of desired steps. In other words, use the formula $(4/3)^N$ where N is the step number. In Sec.2, the results are given in cents scale.

Use of Cent Scale: Instead of successively multiplying a given ratio by itself, one can use cents scale. In cent scale, the operation of successive multiplication is replaced by successive addition and one doesn't have to deal with fractions. Cent scale is a logarithmic scale to base 10. The relative frequency in cent is the logarithm of the relative frequency in the ratio form. A ratio of 2 corresponding to an octave is made equal to 1200 cent.

Relative Frequency in cent = 3986.3 log (Relative Frequency as a ratio).

Madhyama in cent = 3986.3 log(4/3) = 3986.3X0.1249 = 498 cent.
Panchama in cent = 3986.3 log(3/2) = 3986.3X0.17609 = 702 cent.
Tara Sadja in cent = 3986.3 log(2) = 1200 cent.

Cycle of Fourths in Cents Scale: Consider *Panchama* which is 702 cent. *Panchama* of *Panchama* is (702+702) = (1404 cent). Now the relative frequencies in cent are **added**. The result can be expressed as (1200 + 204) cent. Since 1200 cent corresponds to an octave, transposing the note to the primary octave gives a new derived note as 204 cent, one of the 22 *srutis*, viz., *Chatusruti Rishabha*. In the next step, we get (3 times 702) = (1404+702) = (1200+906) or 906 cent. Thus the new note derived is 906 cent, one of the 22 *srutis*, viz., *Chatusruti Dhaivata*. This procedure can be repeated to derive the various *srutis*.

Cycle of Fifths in Cent Scale: Consider *Sudha Madhyama swara* which is 498 cent. *Madhama* of *Madhyama* is (498+498) = 996 cent which is a derived note, one of the 22 *srutis*,viz., *Kaishiki Nishada*. In the next step, we get (498+498+498) = 1494 = (1200+294) or 294 cent, a derived note, one of the 22

srutis, viz., *Sadharana Gandhara*. This procedure can be repeated to derive the various *srutis*.

2. B. Results and Discussion on the Existing Method of Cycle of Fifths and Fourths

The derived values for the *srutis* using the existing method of the cycle of Fifths in cent scale are shown in Table. I. The derived values for the *srutis* using the existing method cycle of Fourths are shown in Table. II.

The *srutis* derived by the application of the cycle of Fifths for the steps 1-11 correspond to 11 *srutis* that are traditionally accepted values. Thus 22 *srutis* are derived. To this extent the method is useful.

At the twelfth step, one expects the results to be either 1200 or 0. However, this doesn't happen. The result at the twelfth step is non-zero, which we refer to as the 'closing error in the octave'. Should one consider 24 or 176 cent as a valid *sruti*?

The cycle of Fifths and Fourths can be applied beyond 12 steps. As a consequence it results in intervals, which do not correspond to the traditionally recognized *srutis*. The results obtained after the eleventh step in both the cycle of Fifths and Fourths are to be discarded. No reason or justification is offered for discarding these derived notes. This is a major drawback of the existing method.

3. Modified Cycle of Fifths and Fourths

Modified Cycle of Fifths in Cent Scale: Consider *Panchama swara, which* is 702 cent. In the modified approach, **24 cent is SUBTRACTED** from the result before proceeding to the next step. Thus (702-24)=678 cent is one of the 22 *srutis*. *Panchama* of *Panchama* is (702+702) = (1404 cent). This results in (1404-24) = 1380 cent = (1200 + 180) cent. 180 cent corresponds to one of the 22 *srutis*. In the next step, one gets (1404+702) = (2106). SUBTRACTING 24 cent yields: (2106-24) = (2082) = (1200 + 882). 882 cent corresponds to one of the 22 *srutis*. This procedure is repeated. The results are shown in Table. III. It may be

seen that with the modified approach, the derived value of the *sruti* at the twelfth step is 1200 or zero cent.

Why should one subtract 24 cents? See Appendix-B. It may be noted that the initial eleven notes derived by the existing method of the cycle of Fifths correctly predict the *srutis*. This means that *differences* between successive relative frequencies of the notes are correct and are to be preserved. Since the twelfth step has an error of 24 cent, it is necessary to remove 24 cents for the steps 1 to 11. Luckily, removing 24 cent from each entry still results in valid *srutis* while preserving the relative frequencies.

If the procedure is to be repeated beyond twelve steps, then the rule is to subtract 48 cent for steps 13-24; subtract 72 cent for steps 25-37; and so on. Then the results already obtained in steps 1-11 repeat themselves. Intervals that are to be discarded are not derived at all.

Modified Cycle of Fourths in Cents Scale: Consider *Sudha Madhyama swara* which is 498 cent. In the modified approach, **24 cent is ADDED** to this result before proceeding to the next step. Thus (498+24)=522 cent corresponds to one of the 22 srutis. *Madhama* of *Madhyama* is (498+498) = 996 cent. Applying the modified rule one gets (996+24) = 1020. Hence 1020 cent corresponds to one of the 22 srutis. In the next step, we get (996+498) = 1494. Now ADD 24 cent. (1200+294+24) to get 318 cents, one of the 22 *srutis*. This procedure is repeated. The results are shown in Table. IV. It may be seen that with the modified approach, the result at the twelfth step happens to be 1200 or zero.

Why should one add 24 cents? See Appendix-B. The logic for adding 24 cent in the cycle of fourths is similar to one given for subtracting 24 cent in the cycle of fifths. If the procedure is to be repeated beyond twelve steps, then the rule is to add 48 cent for steps 13-24; add 72 cent for steps 25-37; and so on. Then the results already obtained in steps 1-11 repeat themselves. No new values that are to be discarded are derived.

Modified Cycle of Fifths in Ratio Scale: In case one works with ratios, then *Panchama* of *Panchama* is (3/2)(3/2) =(9/4). This ratio has to be **REDUCED by (80/81)**. That is multiply (9/4) with (80/81) to yield (2 times 10/9). Hence (10/9) corresponds to one of the 22 *srutis*. The next step results in (3/2)(3/2)(3/2) (80/81) = (10/3) = (2 times 5/3). Hence 5/3 corresponds to one of the 22 *srutis*.

Modified Cycle of Fourths in Ratio Scale: In case one works with ratios, then *Madhyama* of *Madhyama* is (4/3)(4/3) =(16/9). This ratio has to be **INCREASED by (81/80)**. That is multiply (16/9) with (81/80) to yield (9/5). Hence the *sruti* of the derived note corresponds to (9/5). The next step results in (4/3)(4/3)(4/3) (81/80) = (12/5) = (2 times 6/5). Thus the next *sruti* is (6/5).

4. Certain Complementarities Amongst 22 sruits

a) For steps 1-11, the *srutis* derived by the modified method by the cycle of fifths happens to be the *srutis* derived by the existing method of fourths and they occur *in reverse order*. Compare Column 3 of Table-III with Column 2 of Table-II (in reverse order). Similarly, for steps 1-11, the *srutis* derived by the modified method by the cycle of Fourths happens to be the *srutis* derived by the existing method of Fifths and they occur *in reverse order*. Compare Column 3 of Table-IV with Column 2 of Table-I (in reverse order).

b) Certain complementarities amongst 22 sruits are shown in Table.V.A. Sruti derived by modified method of Fourths at any given step can be obtained as the difference between 1200 and the Sruti derived at the corresponding step by the modified method of Fifths. This means if we know 11 srutis derived using modified cycle of Fifths then the other 11 srutis of the cycle of Fourths can be obtained, vice-versa. Hence it is adequate to use either the modified Cycle of Fifths or modified Cycle of Fourths and not both as being done at present.

c) Let us compare the *srutis* derived by the modified cycle of Fifths with the reverse ordered *srutis* derived by the modified cycle of Fourths. The differences are 24 cents at each step. See Table.V.B. This means if we know 11 srutis derived using modified cycle of Fifths then the other 11 srutis of the cycle of Fourths can be obtained, vice-versa. Hence it is adequate to use either the modified cycle of Fifths or the modified cycle of Fourths and not both as being done at present.

d) Of the 22 *srutis*, only 11 *srutis* are presently being widely used in Karnatic music. These are seen in Coulmn 2 of Table VI. Of the eleven *srutis*, only six are independent. For example (Freq of note 11 = 1200 – Frequency of Note 1). Such a symmetry can be interpreted as follows" Rishabha is as far from Aadhara Sadja as Tara Sadja is from Nishada", similarly for other swaras. 612 cents corresponding to *Prati Madhyama* stands alone and has no complimentary note amongst the 11 srutis.

e) Given an entry derived from the modified cycle of Fifths, one can derive three additional entries. Select the first six entries from Table-III and put them in Col.1. The other intervals can be derived as shown below.

Given	**obtain 1200 - col. 1**	**col. 1 + 24**	**1200 - col. 3**
678	522	702	498
180	1020	204	996
882	318	906	294
384	816	408	792
1086	114	1110	90
588	612	612	588

There are 6X4=24 entries. The pair 588, 612 repeats itself. Re-writing the above Table with last column as reference and arranging the notes in an ascending order one gets

Given	obtain 1200 - col. 1	(col. 2 - 24)	1200 - col. 3
90	1110	1086	114
294	906	882	318
498	702	678	522
588	**612**	588	612
792	**408**	384	816
996	**204**	180	1020

New Method of Deriving Srutis:

In the above Table, given any entry in each row, all the other entries can be found. Suddha Madhyma (498 cent) and Prati Madhyama (612) never occur in the same raga as such may be considered as independent. Also, Panchama (702 cent) is a fixed and independent swara. The difference between Panchama and Suddha Madhyama = 204 cent. Given Suddha Madhyma (498) and Prati Madhyama (612) and a value 204, then all the 22 srutis can be obtained. This can be demonstrated by deriving at least one entry in each row in the above Table. 498-204=294; 294-204=90;612-204=408;408-204=204.

5. Conclusion

The author has proposed a modified method of the cycle of Fifths and Fourths and shown that by modifying the rule, the ‘closing error in the octave’ is eliminated and exactly 22 *srutis* are derived. Further it is shown that it is adequate to use either the modified cycle of Fifths or the modified cycle of Fourths and not both. Also, certain hidden complementarities amongst 22 srutis have been brought out explicitly.

6. Acknowledgement

I want to make some personal remarks as a background for this study. Prof R Satyanarayana, a well known musicologist, has been prompting me to work on the so called '*sruti'* problem of Karnatic music. Dr Satyanarana, with his expertise, has suggested replication of the original experiment based on two stringed instruments for deriving the *srutis*. This replication experiment is yet to be done.

In an informal discussion I mentioned to Dr Varadarangan, a renowned Karnatic vocalist, a problem related to the Geometry of Srichakra. By following the traditional approach of drawing Srichakra, at the completion of the drawing, instead of three lines meeting at a point, a small triangle is produced which we called as the closing error. This problem of drawing Srichakra has been discussed in my joint work with Mr Ramakrishna [6]. In that context, Dr Varadarangan pointed out to me a similar problem that exists in musicology which is related to the cycle of Fifths and Fourths. This also prompted the author to work on the problem of calculating the intervals in Karnatic Musical scale.

The author wishes to thank Dr R Satyanarayana and Dr Varadarangan for inspiring me to work on the problem.

Note 1: '*Sruti Bheda'* is yet another method to derive the *srutis*. '*Sruti Bheda'* approach also gives similar results as obtained by the cycle of Fifths and Fourths. The method based on the cycle of Fifths and Fourths is much simpler. The present paper is related to the cycle of fifths and fourths and not '*Sruti Bheda*' method.

Note 2: It is important to mention that the author is an engineer and not a musicologist nor a practicing artist of Karnatic music. The author with his mathematical background has intuitively arrived at some findings that he wishes to place before the critical audience and humbly willing to accept any comments and criticisms on the work.

Note 3: Panchama of Panchama means playing on the Veena, a swara corresponding to 'Panchama fret' on a string that is tuned to Panchama.

References

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2. P. Sambamurthy, South Indian Music, Indian Music Publishing House, Madras, 1981.
3. K. Varadarangan, *Srutis and Srutibheda*, Lalita Prakashana, Bangalore, 2002. Also see K. Varadarangan, "Determination of the relative frequencies and the fundamental properties of swaras using modal shift of tonic", vol. LXXI, J. of Music Academy of Madras, 2000. pp. 66-73.
4. http://cnx.rice.edu/content/latest
5. http://www.perfectthird.com
6. T V Ananthapadmanabha and M N Ramakrishna, "Geometry of Sricakra', Anugraha Publihsers, Bangalore, 2002.

Table. I.

Derived Relative Frequencies in cent by the

Application of the Existing Method of the Cycle of Fifths

Step Number N	Relative Frequency in Cent (702 X N) or Entry in Column 3 + 702.	Relative frequency in cent reduced to Primary Octave. Subtract integral multiples of 1200 if required.
1	702	702
2	1404 (702+702)	204 (1200+204)
3	2106 (or 204+702)	906
4	2808 (or 906+702)	408
5	3510 (or 408+702)	1110
6	4212 (or 1110+702)	612
7	4914 (or 612+702)	114
8	5616 (or 114+702)	816
9	6318 (or 816+702)	318
10	7020 (or 318+702)	1020
11	7722 (or 1020+702)	522
12	**8424 (or 522+702)**	**24 CLOSING ERROR IN OCTAVE** (Not a valid *sruti*)
13	9126 (or 24+702)	726 (Not a valid *sruti*)
14	9828 (or 726+702)	228 (Not a valid *sruti*)
15	10530 (or 228+702)	930 (Not a valid *sruti*)
16	11232 (or 930+702)	432 (Not a valid *sruti*)
17	11934 (or 432+702)	1134 (Not a valid *sruti*)

… and so on.

Table. II.

Derived Relative Frequencies in cent by the

Application of the existing method of the Cycle of Fourths

Step Number N	Relative Frequency in Cent (498 X N) or Entry in Column 3 + 498.	Relative frequency reduced to Primary Octave. Subtract integral multiples of 1200 if required.
1	498	498
2	996 (or 498 + 498)	996
3	1494 (or 996+498)	294 (1200 + 294)
4	1992 (or 294+498)	792
5	2490 (or 792+498)	90
6	2988 (or 90+498)	588
7	3486 (or 588+498)	1086
8	3984 (or 1086+498)	384
9	4482 (or 384+498)	882
10	4980 (or 882+498)	180
11	5478 (or 180+498)	678
12	**5976 (or 678+498)**	**1176 =1200-24. CLOSING ERROR IN OCTAVE** (Not a valid *sruti*)
13	6474 (or 1176+498)	474 (Not a valid *sruti*)
14	6972 (or 474+498)	972 (Not a valid *sruti*)
15	7470 (or 972+498)	270 (Not a valid *sruti*)
16	7968 (or 270+498)	768 (Not a valid *sruti*)
17	8466 (or 768+498)	1062 (Not a valid *sruti*)

… and so on.

Table. III.

Derived Relative Frequencies in cent by the Application of the Modified Cycle of Fifths as Proposed by Ananthapadmanabha

Step Number N	Relative Frequency in Cent (702 X N)	**SUBTRACT** 24 cent for steps 1-12; 48 cent for steps 13-24 and so on.	Relative frequency reduced to Primary Octave. Subtract integral multiples of 1200 if required.
1	702	678	678
2	1404	1380	180 (1200+180)
3	2106	2082	882
4	2808	2784	384
5	3510	3486	1086
6	4212	4188	588
7	4914	4890	90
8	5616	5592	792
9	6318	6294	294
10	7020	6996	996
11	7722	7698	498
12	**8424**	**8400**	**0. No Closing Error**
13	9126	9078	678 (same as in step 1)
14	9828	9804	180 (same as in step 2)

Table. IV.

Derived Relative Frequencies in cent by the Application of the Modified Cycle of Fourths as Proposed by Ananthapadmanabha

Step Number N	Relative Frequency in Cent (702 X N)	**ADD** 24 cent for steps 1-12; 48 cent for steps 13-24; and so on	Relative frequency reduced to Primary Octave. Subtract integral multiples of 1200 if required.
1	498	522	522
2	996	1020	1020
3	1494	1518	318 (1200+318)
4	1992	2016	816
5	2490	2514	114
6	2988	3012	612
7	3486	3510	1110
8	3984	4008	408
9	4482	4506	906
10	4980	5004	204
11	5478	5502	702
12	**5976**	**6000**	**0. No Closing Error.**
13	6474	6522	522 (same as in step 1)
14	6972	5502	1020 (same as in step 2)

Table. V.A.

Relationship Between the *Srutis* derived using Modified Cycle of Fifths and Fourths

No	*Sruti* obtained by the modified method of Fifths.	*Sruti* obtained by the modified method of Fourths.	Sum of the values in Col. 2 and Col. 3
1	678	522	1200
2	180	1020	1200
3	882	318	1200
4	384	816	1200
5	1086	114	1200
6	588	612	1200
7	090	1110	1200
8	792	408	1200
9	294	906	1200
10	996	204	1200
11	498	702	1200

Table. V.B

Relationship Between the *Srutis* derived using

Modified Cycle of Fifths and Fourths

No	*Sruti* obtained by the modified method of Fifths.	*Sruti* obtained by the modified method of Fourths in reverse order.	Difference between Col 3 and Col. 2 (Col.3-Col. 2)
1	678	702	24
2	180	204	24
3	882	906	24
4	384	408	24
5	1086	1110	24
6	588	612	24
7	090	114	24
8	792	816	24
9	294	318	24
10	996	1020	24
11	498	522	24

Table. VI.

Redundancies Amongst the 11 srutis – Ascending Order

No	Notes in ascending order	Redundancy	Successive Differences
0	0	Note 12 = 1200 – 0	(1) –(0) : **90**
1	90	Note 11 = 1200 – 90	(2)-(1) : **114**=90+24
2	204	Note 10 = 1200 – 204	(3)-(2) : **90**
3	294	Note 9 = 1200 – 294	(4)-(3) : **114**=90+24
4	408	Note 8 = 1200 – 408	(5)-(4): **90**
5	498	Note 7 = 1200 – 498	(6)-(5) : **114**=90+24
6	***612***		
Following Srutis are redundant as (7)-(12) are already predicted above.			
7	**702**	**1200 – Note 5**	**(8) – (7) : 90**
8	792	1200 – Note 4	(9)-(8) : **114**
9	906	1200 – Note 3	(10)-(9) : **90**
10	996	1200 – Note 2	(11)-(10) : **114**
11	1110	1200 – Note 1	(1200 cent – 1110) : **90**
12	1200	1200 – Note 0	

Appendix – A

***Adhara Shadja**:* An artist of classical Karnatic music chooses a certain **reference fundamental frequency** for a concert called the *Adhara Shadja*. Throughout the concert this reference fundamental frequency doesn't change. This is also sometimes referred to as 'tonic' note.

Octave: Twice the reference fundamental frequency is said to be one octave above the reference.

Sapta Swaras: A lay man understands that there are seven basic notes or swaras described as 'sa, ri, ga, ma, pa, dha, ni'. The names of these notes are 'Sadja, Rishabha, Gandhara, Madhyama, Panchama, Dhaivata, Nishada'. The eighth swara is one octave of 'sa or sadja' called *tara* shadja.

Eleven Swaras: Except Sadja and Panchama, other swaras have their own twin or triplet associates. There are subtle variations in other swaras. For example there are two Madhyamas: Prati-Madhyama and Anu-Madhyama; there are three Rishabas etc. Based on these subtle variations in the swaras, eleven swaras are identified within an octave. If we include the tara sadja then there are 12 swaras. Of course only a subset of these swaras occur in a given raga.

Interval: The **ratio** of the frequency of a swara relative to the adhara sadja is called an interval. (Not the difference in frequencies.). Why the ratio? This is because the auditory system judges the similarity of two notes based on the ratios of frequencies. Let use want to compare two notes 'X' and 'Y'. If 'X' changes to '1.2X' then 'Y' must change to '1.2Y' then the two notes will be heard as similar. Consider as an example Adhara Sadja of say 250 Hz. Its Panchama will be 1.5 times 250 = 375 Hz. If the Aadhara Sadja changes to 300 Hz then Panchama will change to 1.5 times 300 =450 Hz. Hence we only have to state the ratios of the frequencies of the swaras. These ratios are called 'intervals'.

Sthayi: Let the frequency of Aadhara Sadja be 'F'. Its octave is '2F'. The range of frequencies F to 2F is one octave called 'madhyama sthayi'. The frequency of tara sadja is 2F. Its octave is 4F. Thus the range of the upper octave is 2F to 4F. This is called tara sthayi. All the swaras in the tara sthyai will be exactly twice the frequency of the corresponding swaras in the 'madhayam'

sthayi. In other words the ratio of frequencies of any two swaras within an octave is the same irrespective of the sthayi.

22 Srtuis: Sruti is sometimes defined as the smallest difference which is noticeable and also have a tonal musical quality. Considering the subtle variations in the swaras traditionally 22 srutis are recognized within an octave. In other words, except Sadja, Panchama, Suddha Madhyama and Prati Madhaya, other swars have additional counterparts.

Appendix-B

Significance of Adding or Subtracting 24 Cent in the Modified Method

One has to be cautious regarding the numerical values. If required one can follow rigorous arithmetic calculations at the expense of clarity and easy readability. The author is aware of these issues. For ease of calculation, the value for *Panchama* is used as 702 cent. The actual value is 701.955. If the actual value is used, then the correction of 24 cent suggested to be added or subtracted also has to be modified accordingly.

In ratio scale it was suggested to decrease the ratio by a factor of 80/81 or increase the ratio by a factor of 81/80. These factors are best *rational approximations*. Ideally, for the cycle of fifths, what is desired is, when (3/2) is raised to power 12, it must come close to an integer power of 2. That is one has to solve for the following equation:

$$\{ (3/2)^{12} \} \{ \text{Factor} \} = \{ 1.5^{12} \} \{ \text{Factor} \} = 2^{N}$$

where N is an integer. This Factor comes out as 0.986540368 and N=7. In cents scale, the factor comes close to 24 cents. But the rational number 80/81 = 0.987654321 is closest to the desired value.

Similarly, for the cycle of fourths, when (4/3) is raised to power 12, it must come close to an integer power of 2. That is one has to solve for the following equation:

$$\{ (4/3)^{12} \} \{ \text{Factor} \} = \{ 1.3333^{12} \} \{ \text{Factor} \} = 2^{N}$$

where N is an integer. The Factor once again comes out as 0.986540368 and N=5. In cents scale, the factor comes close to 24 cents. The rational number 80/81 = 0.987654321 is closest to the desired factor.